A Computational Model
of Ratio Decidendi

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Abstract

This paper proposes a model of *ratio decidendi* as a justification structure consisting of a series of reasoning steps, some of which relate abstract predicates to other abstract predicates and some of which relate abstract predicates to specific facts. This model satisfies an important set of characteristics of *ratio decidendi* identified from the jurisprudential literature. In particular, the model shows how the theory under which a case is decided controls its precedential effect. By contrast, a purely exemplar-based model of *ratio decidendi* fails to account for the dependency of precedential effect on the theory of decision.

1 Introduction

A central obstacle to the development of automated legal reasoning systems for common-law jurisdictions is the problem of modeling precedent-based legal reasoning. In common-law jurisdictions, reasoning with precedents is an essential part of every effective attorney's repertory of skills. A computer system intended to emulate human problem-solving behavior in such domains must therefore also be capable of reasoning with precedents.

Any complete account of precedent-based legal reasoning must include a model of *ratio decidendi*, the content of a precedent that is authoritative as to subsequent cases. Predicting, advocating, and justifying the binding effect of a precedent on subsequent cases all require identifying the authoritative elements of the precedent and showing how they can apply to subsequent cases involving different facts.

Development of an adequate computer model entails three distinct tasks. First, the phenomenon to be modeled must be precisely specified. Second, an appropriate computational model must be described. Finally, the ability of the computational model to account for the phenomenon must be demonstrated.

The next section addresses the the first of these tasks—describing the phenomenon to be modeled—arguing that the jurisprudential literature on legal precedent provides a set of criteria for the adequacy of models of *ratio decidendi*. Section three describes a model of *ratio decidendi*, termed the *reduction-graph* model, under which the *ratio decidendi* of a precedent is a justification structure consisting of a series of reasoning steps, some of which relate abstract predicates to other abstract predicates and some of which relate abstract predicates to specific facts. Section four argues that the reduction-graph model satisfies most of the adequacy criteria set forth in section two. Section five argues that a purely exemplar-based model of legal precedent consisting of representations of the material facts of each precedent case together with a global relevance metric fails to satisfy an important

criterion: representing how the theory under which a case is decided controls its precedential effect. Three pragmatic issues in the implementation of the reduction-graph model are discussed in the last section: the difficulty of exemplar matching; the role of the backing of warrants; and the problem that the written justifications for judicial decisions are often incomplete.

The reduction-graph model is intended as a *knowledge level* [Newell, 1982] description of precedent-based reasoning, that is, a "specification of what a reasoning system should be able to do" independent of any particular "symbol-level" implementation of this process. The emphasis of this paper is therefore on identifying the knowledge required for precedent-based reasoning and showing how this knowledge is used in the resolution of new cases.

2 Evaluation Criteria for Models of Ratio Decidendi

The first step in developing a computational model of *ratio decidendi* is specifying the characteristics of *ratio decidendi* that the model should account for. Several knowledge sources for describing the phenomenon of precedent-based legal reasoning can be distinguished. One possible source of information would be detailed empirical studies of judges' and attorneys' use of precedents in problem solving. Unfortunately, few such empirical studies exist.

An alternative source of information is introspection on one's own use of precedents. Unfortunately, introspection is notoriously unreliable [Gardner, 1985]. However, the law's "tradition of examining its processes and assumptions" [Rissland, 1990] as embodied in jurisprudential literature provides another alternative. While jurisprudential writings are often contradictory and frequently serve a prescriptive rather than a descriptive agenda, jurisprudential ideas that survive prolonged critical scrutiny are likely to contain at least an element of truth.

There is little dispute that within the Anglo-American system of law, judicial precedents are a primary source of legal authority [Bodenheimer, 1974]. The authoritative character of precedents is expressed in the term *stare decisis*,[1] which signifies that when a point of law has been settled by a judicial decision, it is not ordinarily to be departed from afterward.

Unfortunately, there is less unanimity about precisely how "the point of law settled by a judicial decision," *i.e.*, the *ratio decidendi* of the decision, is to be determined. A number of different characterizations of *ratio decidendi* have been proposed. These characterizations tend to fall into two general approaches, differing primarily in the degree of generality or abstractness attributed to the *ratio decidendi*.[2] One approach views the *ratio decidendi* of a precedent as a general rule or principle, typically contained in the language of the judge's decision. The emphasis of advocates of this approach is usually on isolating the *ratio decidendi* from surrounding nonauthoritative language, or *dictum*. The alternative approach focuses on the specific facts of precedents. This approach tends to be associated with skepticism about the ostensible justifications advanced by judges for their decisions. An important issue under this approach is determining precisely which facts of a case were material to the judge's decision.

The remainder of this section sets forth five widely recognized characteristics of *ratio decidendi* that can be discerned from among the diversity of jurisprudential proposals.

1. The *ratio decidendi* of a precedent consists of propositions of law, explicit or implicit in the opinion, that are necessary to the decision.

This characterization of ratio is rooted in two observations. The first is that judicial decisions almost invariably contain a written justification. Appellate opinions typically contain a summary of the facts of the case, identification of the issues of law raised in

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[1]*Stare decisis* is an abbreviation for *stare decisis et non quieta movere*, meaning "to stand by precedents and not to disturb settled points."

[2]*See* [Collier, 1988] for a detailed comparison of these two approaches.

arguments by counsel for each of the parties, pronouncement of the legal propositions supported by the controlling authorities, and declaration of a decision that resolves the issues by applying the legal propositions to the facts of the case. Predictability, judicial economy, and the principle that like cases should be treated alike all suggest that the legal propositions under which a case is resolved should apply equally to similar future cases.

The second observation is that not every part of the written justification is authoritative. As early as 1673,[3] it was recognized that written decisions frequently contain language unnecessary for the resolution of the issues before the court and that this unnecessary language is not part of the *ratio decidendi* of the case.

These two observations together suggest that the *ratio decidendi* of a case consists of just those pronouncements of legal propositions by the judge that are necessary to the resolution of the issue before the court. The numerous advocates of this characterization of *ratio decidendi* include Edmund Morgan, who argued that the *ratio decidendi* of a case consists of "those portions of the opinion setting forth the rules of law applied by the court, the application of which was required for the determination of the issues presented ...." [Morgan, 1948]. Similarly, Sir John Salmond stated that courts should limit themselves to formulation of principles that are "required for the due decision of the particular case" [Salmond, 1900].

Another proponent of this view, Eugene Wambaugh, proposed a widely accepted test for determining whether a given proposition is the *ratio decidendi* of a precedent: if the deciding court could have believed the negation of the proposition without changing the outcome of the case, the proposition is *dictum* rather than *ratio decidendi* [Wambaugh, 1894]. Wambaugh's formulation differs somewhat from that of Morgan and Salmond in holding that the proposition representing the *ratio decidendi* of a case might not actually

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[3] *Bole v. Horton*, Vaughn 360.

be articulated by the court. However, Wambaugh asserted that it can be discovered by anyone who "diligently studies the problem and the result" [Wambaugh, 1894].

2. A unique proposition of law necessary to a decision can seldom be determined. Instead, a gradation of propositions ranging in abstraction from the specific facts of the case to abstract rules can satisfy this condition.

A widespread but naive view of *ratio decidendi* is that the *ratio decidendi* of a precedent consists of a single, unique proposition of law "without which the case must have been decided otherwise." Rupert Cross in "Precedent in English Law" [Cross, 1979] illustrated the untenability of this view with the example of *Donoghue v. Stevenson*,[4] a case holding the manufacturer of a bottle of ginger beer containing a decomposed snail liable to the ultimate consumer. Lord Atkin's opinion contained two propositions of law justifying the decision. The first was very general:

> A party must take reasonable care to avoid acts or omissions which he can reasonably foresee would be likely to injure persons ... closely and directly affected by his act ....

The second was much more specific:

> A manufacturer of products, which he sells in such a form as to show that he intends them to reach the ultimate consumer in the form in which they left him with no reasonably possibility of intermediate examination ... owes a duty to the consumer to take ... reasonable care.

Cross observes that both of these propositions satisfy Wambaugh's test: the negation of either proposition would require a different decision.

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[4]A.C. 562 (1932).

Similarly, Herman Oliphant in *A Return to Stare Decisis* [Oliphant, 1928] (as paraphrased in [Collier, 1988]) analogized the process of viewing a precedent to that of a spectator entering a stadium:

> The spectator has a choice not only of where around the field to sit (angle of view, or legal subject area), but also of how far up the rows of bleachers to go (level of generalization). No internal logic dictates a resting place in either dimension.

Oliphant gave the example of a decision holding that a father was privileged to induce his daughter to break a promise to marry. Oliphant argued that the decision was consistent with any of six distinct holdings, ranging in generality from "1. Fathers are privileged to induce daughters to break promises to marry" to "6. All persons are so privileged as to all promises made by anyone."

3. The *ratio decidendi* of a precedent must be grounded in the specific facts of the case.

An important source of uncertainty in legal reasoning is the "gap" in generality between abstract legal concepts and the specific facts of new cases [Gardner, 1987, Branting and Porter, 1991]. Precedents are useful in deciding new cases because they provide examples of specific facts that satisfy such concepts. Accordingly, a model of *ratio decidendi* that omits specific case facts cannot fully account for the precedential effect of past cases.

Typical expressions of the centrality of precedents' specific facts include:

- "[I]t is the facts, and not the general rules of law found in precedents ... which serve as the foundation of the decisional process." [Cueto-Rua, 1981] at 56.

- "Judgements must be read in light of the facts of the case in which they are delivered." [Cross, 1979] at 44.

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- "It is clear that the most important part of a representation for cases is the representation of the facts and the outcome." [Gardner, 1987] at 47. Gardner quotes with approval Corbin's admonition that cases should be studied "not so much for their doctrinal statements as for ... their 'operative facts' " and Gilmore's statement that one should "never quote general language from an opinion, divorced from the factual context in which the language was delivered."

Some legal scholars have taken the extreme position that the specific facts of precedents are not just *necessary*, but are also *sufficient* representations of *ratio decidendi*. Early proponents of this view included adherents of the *legal realist* movement. Legal realists adopted an empirical approach to *ratio decidendi* because they were skeptical about the importance of judges' ostensible justifications in judicial decision making. For example, Oliphant opined that:

> the predictable element ... is what courts have done in response to the stimuli
> of the facts of the concrete cases before them. Not the judges' opinions, but
> which way they decide cases, will be the dominant subject matter of any truly
> scientific study of law [Oliphant, 1928].

In its most extreme form, the realist position held that judges' written explanations have no bearing whatever on the outcome of cases and that cases represent only judges' disposition to behavior in response to stimuli [Collier, 1988]. This position has few adherents today.[5]

A more tenable formulation of this position was put forth by Arthur Goodhart. Goodhart argued that "the *ratio decidendi* of a case ... must not be sought in the reasons on which the judge has based his decision" but must be sought instead in "the material facts

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[5]However, *jurimetrics* represents an attempt to derive useful empirical information concerning judges' behavior without reference to their written justifications.

on which the judge has based his conclusion" [Goodhart, 1930]. Goodhart asserted that a judge:

> founds his conclusions upon a group of facts selected by him as material from among a larger mass of facts, some of which might seem significant to a layman, but which, to a lawyer, are irrelevant. ...It follows that our task in analyzing a case is ...to state the material facts as seen by the judge and his conclusion based on them. It is by his choice of the material facts that the judge creates law [Goodhart, 1930].

Goodhart shared the skepticism of Oliphant and the other realists that the *ratio decidendi* of a precedent is to be found in judges' enunciation of legal rules, but differed from the realists in believing that it nevertheless is possible to determine the *ratio decidendi* of a case from the judge's written opinion:

> [T]he reasons given by the judge in his opinion, or his statement of the rule of law which he is following, are of peculiar importance, for they may furnish us with a guide for determining which facts he considered material and which immaterial. His reason may be incorrect and his statement of the law too wide, but they will indicate to us on what facts he reached his conclusion. [Goodhart, 1930] pp. 175–176.

4. The *ratio decidendi* of a precedent includes not only the precedent's material facts and decision, but also the theory under which the material facts lead to the decision.

Goodhart's model of *ratio decidendi* as material facts plus decision thereon has now been largely rejected. The most important reason for this rejection is that:

[sometimes] it is quite impossible to formulate the *ratio decidendi* merely by reference to the facts, regarded as material by the court, and the decision based on those facts. It is often essential to know why certain facts were regarded as material and for this purpose it may be necessary to know what portions of the law were in the mind of the court when the selection was made. [Cross, 1979] at 73.

In illustrating the importance of knowing the theory under which a case was decided, Cross gives the example of *Bourhill v. Young*,[6] in which the following facts were found to be material:

> Young, a motorcyclist, was killed because of his own negligence when he passed a tram at excessive speed and collided with a car about 50 feet beyond the tram. At the time of the accident, the tram was stopped and Mrs. Bourhill was alighting. Mrs. Bourhill heard the collision and saw blood on the road after the accident and as a result suffered a nervous shock. Mrs. Bourhill was outside what Young ought to have contemplated as the area of potential danger that would arise from his careless driving.

The decision was that Mrs. Bourhill's action against Young's estate was dismissed.

From the material facts and the decision alone it is impossible to determine which of the following two rationales underlies the decision: (1) a driver owes no duty of care in respect of his driving to persons outside the area of reasonably foreseeable danger, or (2) although the driver owes a duty of care to such persons, damages flowing from nervous shock are too remote a consequence of the breach of duty to be recoverable.

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[6]A.C. 92 (1943).

Without knowledge of the controlling rationale, it is impossible to determine from the material facts and the decision alone how either of the following hypotheticals should be decided:

- **H1.** The same facts as *Bourhill* except that the motorcycle driven by Young collides with a fireworks truck instead of a car, and Mrs. Bourhill is burned by a fragment from the resultant explosion of fireworks.

- **H2.** The same facts as *Bourhill* except that Young passes the tram on the same side as the alighting Mrs. Bourhill, missing her by inches and causing a severe emotional shock.

Under the first rationale, Mrs. Bourhill could recover in *H2* but not *H1*. Under the second rationale, the results would be reversed.

To summarize, in some cases, such as *Bourhill v. Young,* there may be alternative theories under which the material facts could have led to the decision. A representation of *ratio decidendi* that includes only the material facts but excludes the operative theory is insufficient to distinguish among these theories.

5. Subsequent decisions can limit, extend, or overturn earlier precedents.

The effect of later decisions on earlier decisions has led many scholars to argue that the *ratio decidendi* of a case is determined, at least in part, by subsequent cases. According to Sidney Post Simpson and Julius Stone (as summarized in [Bodenheimer, 1974]):

> [E]ach case has implicit in it a whole congeries of possible principles of decision. When a case is decided, no one can be certain which of the possible principles of decision is destined eventually to become the controlling one . . . . Only a study of a whole series of decisions on a particular problem of law will to some extent reveal what the fate of a particular precedent has been in the dynamic process

of restricting, expanding, interpreting, reinterpreting, and reformulating a prior body of doctrine ....

There are innumerable instances in which the exact scope of a precedent is gradually defined by subsequent cases. Most commonly this occurs when the language of an earlier opinion is narrowed and circumscribed by subsequent cases. Goodhart [Goodhart, 1930] discusses the example of *Rex v. Fenton*[7] which contained the following language: "If death ensues as the consequence of a wrongful act, an act which the party who commits it can neither justify nor excuse, it is not accidental death, but manslaughter." Fifty-three years later, *Regina v. Franklin*[8] held that " a civil wrong ...is immaterial to this charge of manslaughter." *Regina* thus narrowed the scope of *Rex* by restricting the wrongful acts that can give rise to manslaughter to criminal, as opposed to merely civil, wrongs.

Similarly, the scope of a precedent can be expanded by subsequent opinions. For example, in *Barwick v. English Joint Stock Bank*[9] Willes, J. ruled that "The general rule is, that the master is answerable for every such wrong of the servant or agent as is committed in the course of the service and for the master's benefit, though no express command or privity be proved." Forty-five years after this decision, the English House of Lords ruled in *Lloyd v. Grace, Smith & Co.*[10] that the words "and for the master's benefit" were merely descriptive of the facts in the *Barwick* case, and not a necessary part of the principle involved.

Much of the law concerning the compensability under worker's compensation law for injuries occurring while traveling consists of accumulated exceptions to the common-law rule that commuting is not an activity within the scope of employment [Larsen, 1989]. Similarly, much of the law of products liability consists of incremental extensions to an

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[7] 1 Lew. C. C. 179 (1830).

[8] 15 Cox C. C. 163 (1883).

[9] L. R. 2 Ex. 259 (1867).

[10] A. C. 716 (1912).

initially narrow common-law rule [Levi, 1949].

In consonance with these examples, Goodhart observed that:

> some precedents will always remain indeterminate . . . [F]urther decisions are frequently required before the scope of a principle is finally determined . . . Paton's metaphor is a sound one: 'One case, so to speak, plots a point on the graph of tort, but to draw the curve of the law we need a series of points.'

An adequate model of precedent should, at a minimum, account for these five recognized characteristics of *ratio decidendi*. Accordingly, these characteristics can provide a set of evaluation criteria for models of precedent-based reasoning.

3   The Reduction Graph Model of Ratio Decidendi

This section proposes a model of *ratio decidendi* intended to satisfy the criteria set forth in the previous section. The central tenet of this model is that the *ratio decidendi* of a precedent is best modeled as a justification structure consisting of a series of reasoning steps, some of which relate abstract predicates to other abstract predicates and some of which relate abstract predicates to specific facts. Inference steps of the first type are rules; those of the second type are termed *exemplars*. The authoritative elements of the precedent are the rules and exemplars constituting the justification for the decision.

3.1   Rules and Exemplars as Warrants

A rule is an expression of the connection between a set of conditions, or *antecedents*, and a conclusion, or *consequent*, justified by those conditions. For example, legal rules express the connection between legal predicates and conditions that can satisfy those predicates. In the

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following legal rule, the consequent is the legal predicate *negligence liability* and the three numbered antecedents are the conditions under which the negligence liability predicate is satisfied:

> A person is liable to another person for damages under negligence if (1) the first person owed a duty of reasonable care to the second, (2) the first person breached that duty, and (3) the breach was the proximate cause of the damages suffered by the second person.

An exemplar is a collection of facts to which a conclusion is known to apply. Thus, an exemplar also expresses a connection between a set of conditions and a conclusion justified by those conditions. For example, an exemplar of negligence might be the following:

> Dr. Jones failed to exercise reasonable medical care by counting sponges during surgery on Brown. As a result, a sponge was left in Brown, who developed peritonitis and required a second operation to remove the sponge.

The essential difference between rules and exemplars lies in the degree of generality of the antecedent conditions that justify the conclusion. In rules, these conditions are typically abstract, *e.g.,* "proximate cause," "duty of reasonable care," whereas in exemplars the conditions are very specific, *e.g.,* "Jones left a sponge in Brown's surgical incision." The language in which the concrete, specific, observable facts of a particular case are expressed is termed the *case-description* language. By contrast, the abstract terms that appear in rule antecedents but that do not appear in the case-description language are termed *abstract features* [Porter et al., 1990].

Although they differ in the generality of their conditions, rules and exemplars are alike in that each can be used to justify a conclusion in a new case that matches the conditions. If the connection between the conditions and the conclusion reflects a regularity within the

domain, the same conclusion is justified in any new case that matches the same conditions. Thus, both rules and exemplars can act as *warrants* [Toulmin, 1958] for conclusions about new cases.

Warrants can be expressed at a variety of levels of abstraction. For example, warrants for negligence include the following:

- General rules (*e.g.,* an action is negligent if the actor fails to use reasonable care and the failure is the proximate cause of damages)

- Specific exemplars (*e.g.,* Dr. Jones was negligent because he left a sponge in Brown during surgery)

- Warrants at intermediate levels of abstraction (*e.g.,* a medical procedure is negligent if it fails to conform to the standard of reasonable care of the medical community and thereby injures a patient)

The relationship between warrants at different levels of generality is expressed by *reduction operators.* Each reduction operator expresses a taxonomic relationship between the antecedents of different warrants for the same conclusion (*e.g.,* breach of the standard of reasonable care of a medical community is a kind of failure to use reasonable care). A hierarchy of warrants for a given conclusion and the reduction operators that connect them is termed a *warrant hierarchy.* Figure 1 illustrates a portion of a warrant hierarchy for negligence. The least abstract warrants in such a hierarchy are exemplars.

Reduction operators are themselves warrants, since they express the connection between an abstract feature and a set of conditions under which the abstract feature is satisfied. In particular, reduction operators that justify conclusions in terms of facts expressed in the case-description language (*e.g.,* failure to count sponges is a failure to conform to the standard of reasonable care of a medical community) are exemplars.
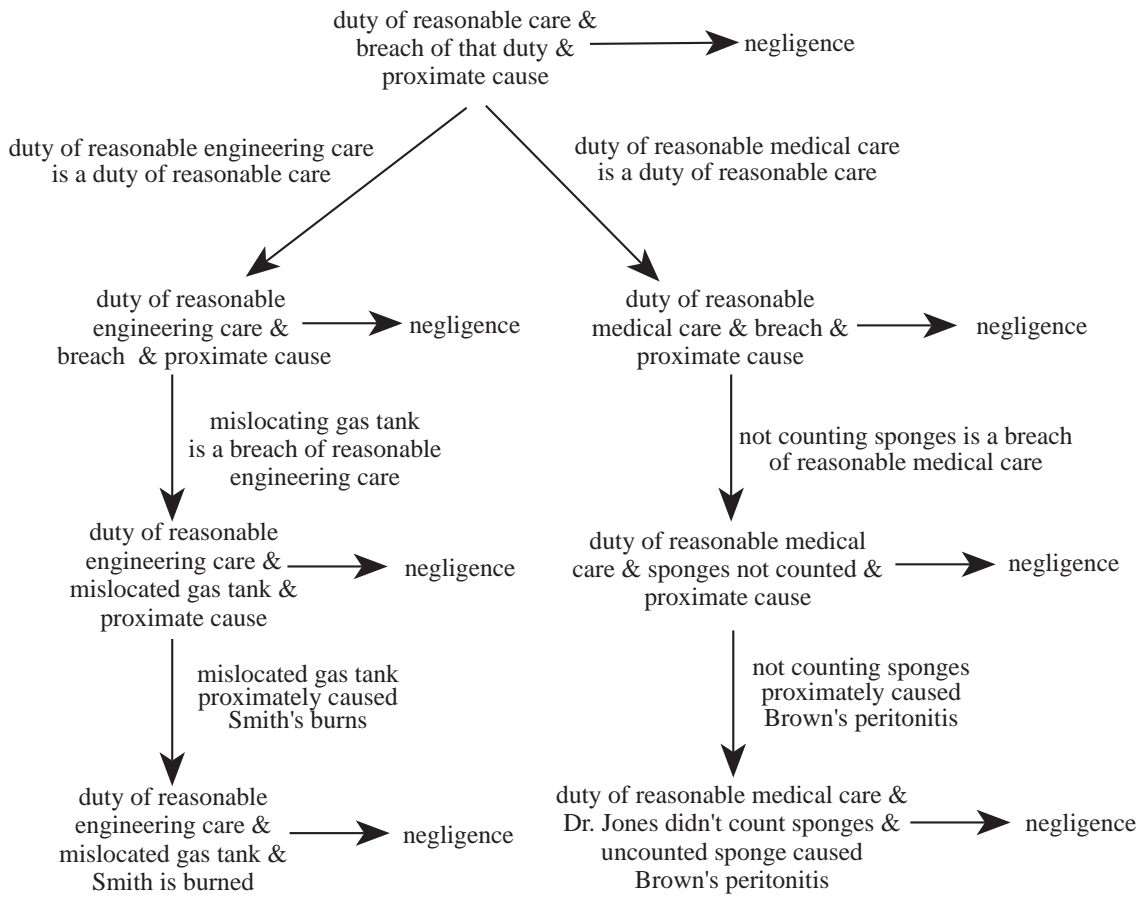
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Figure 1: A warrant hierarchy consisting of four levels of warrants for negligence. Vertical arrows represent reduction operators expressing a taxonomic relationship between facts of warrants on different levels, *e.g.,* failing to count sponges during an operation is a kind of breach of reasonable medical care; the connection between failing to count sponges and peritonitis caused by an uncounted sponge is a kind of proximate cause.

Establishing a conclusion about a new case requires matching the facts of the new case with the facts of some authoritative warrant for that conclusion. For example, if the authoritative pronouncements on negligence in a given jurisdiction were just those warrants shown in figure 1, establishing negligence in a new case would require matching the antecedents of one of the warrants to the facts of the new case.

Matching the antecedents of a warrant to the facts of a new case typically requires inference. Warrants other than exemplars contain abstract features (*e.g.*, reasonable medical care) in their antecedents. Matching the abstract features to the specific facts of a new case (*e.g.*, failure to count the sponges used during surgery) requires reduction operators to bridge the gap between the abstract features and the specific facts (*e.g.*, the reduction operator that identifies failure to count the sponges used during surgery as a kind of failure to conform to the standard of reasonable medical care). Similarly, matching the facts of an exemplar to the facts of a new case may require inference to establish the equivalence of different facts (*e.g.*, failure to monitor blood oxygen level is equivalent to failure to count the sponges used during surgery because both are failures to conform to the standard of reasonable care of the medical community).

A justification for the conclusion that a predicate applies to a case consists of a warrant for the predicate together with all inferences necessary to match the antecedents of the warrant to the facts of the case. Various representations of such a justification are possible. Figure 2 represents a justification in terms of the subgoals that arise in the process of constructing an inference path from the predicate to be established to the facts of a case. The predicate to be established, negligence, is at the top. The rule that duty of reasonable care, breach of that duty, and proximately caused harm imply negligence permits this goal to be reduced to the subgoals of establishing a duty of reasonable care, breach of that duty, and proximately caused harm. These subgoals can in turn be reduced by reduction
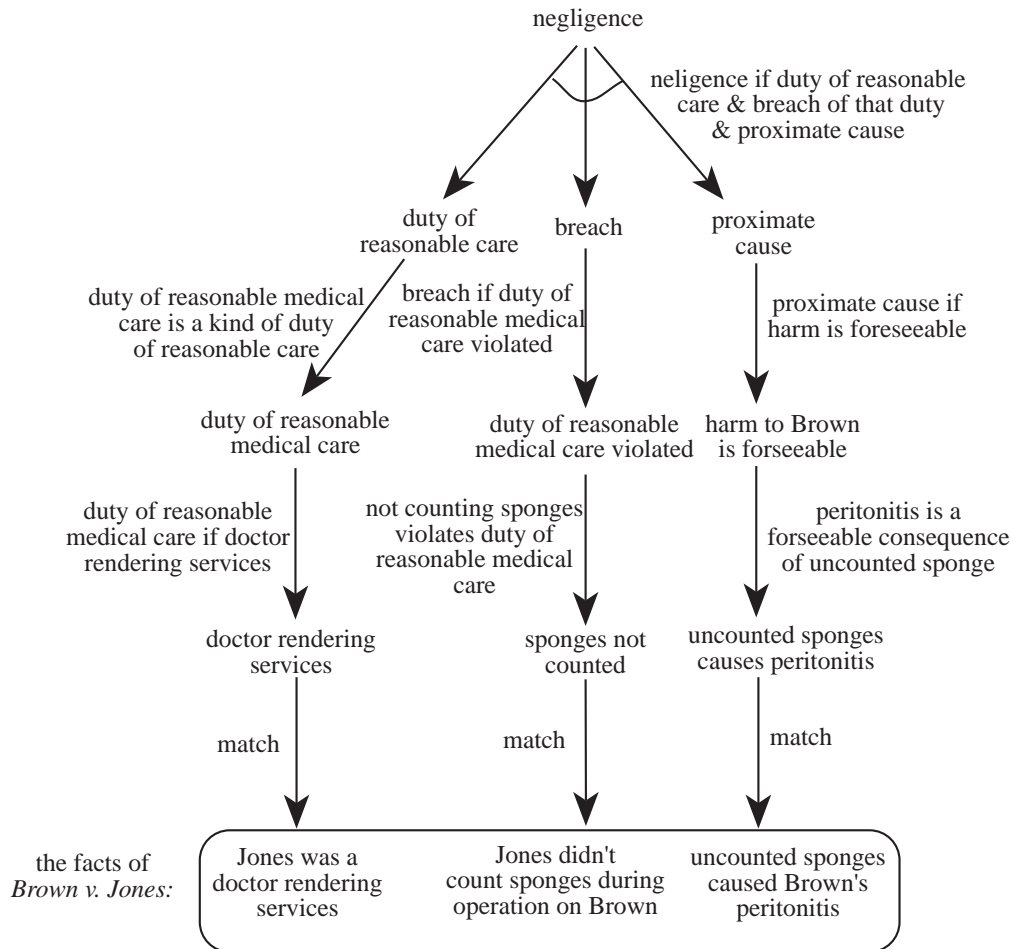
Figure 2: The justification, represented as a goal-reduction graph, for Jones' liability to Brown under negligence for the injuries that resulted from a sponge left by Jones in Brown's abdominal cavity during an operation.

operators to the facts of the case: that Jones was a doctor rendering professional services in operating on Brown, that Jones failed to count sponges during the operation, and that Brown developed peritonitis from the sponge left during the operation. This representation is termed a *goal-reduction graph*.[11]

Figure 3 shows an equivalent alternative representation of this justification in which reduction operators are used to repeatedly rewrite the antecedents of a warrant for negligence until it matches the facts of the case. This representation is referred to as a *warrant-reduction graph*. Goal-reduction graphs and warrant-reduction graphs are simply alternative representations of an identical reduction graph [Amarel, 1968]. The two representations differ only in that the order of reduction operator application and the resulting warrants for the ultimate result are made explicit in a warrant-reduction graph, whereas the subgoals addressed by each reduction operator are made explicit in the goal-reduction graph.

3.2   The Elements of Ratio Decidendi

A judicial decision consists of a determination that some legal predicate, *e.g.*, negligence liability, is satisfied by the facts of a case as determined by the trier of fact. If the decision has a justification in terms of these facts, this justification must necessarily include a warrant for the predicate together with all inferences necessary to match the antecedents of the warrant to the facts. Predictability, judicial economy, and the principle that like cases should be treated alike all argue that each warrant in this justification should apply equally to similar future cases.

In figure 3, for example, the warrants necessary for the ultimate conclusion that negligence liability applies to Jones include (1) each warrant for negligence appearing in the

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[11]This representation is also referred to as an *AND/OR graph* [Barr and Feigenbaum, 1982] or an *explanation structure.* [Mooney, 1988] at 14.

duty of reasonable care &
breach of that duty &
proximate cause $\longrightarrow$ negligence

duty of reasonable medical
care is a kind of duty
of reasonable care

duty of reasonable
medical care &
breach of that duty &
proximate cause $\longrightarrow$ negligence

proximate cause if
harm is foreseeable

duty of reasonable
medical care &
breach of that duty &
foreseeable harm $\longrightarrow$ negligence

not counting sponges
violates duty of
reasonable medical
care

duty of reasonable
medical care &
not counting sponges &
foreseeable harm $\longrightarrow$ negligence

peritonitis is a
forseeable consequence
of uncounted sponge

duty of reasonable
medical care &
not counting sponges &
peritonitis results $\longrightarrow$ negligence

duty of reasonable
medical care if doctor
rendering services

doctor rendering services &
not counting sponges &
peritonitis results $\longrightarrow$ negligence

match

the facts of
Brown v. Jones:
Jones was a doctor rendering services &
Jones didn't count sponges during
operation on Brown & an uncounted
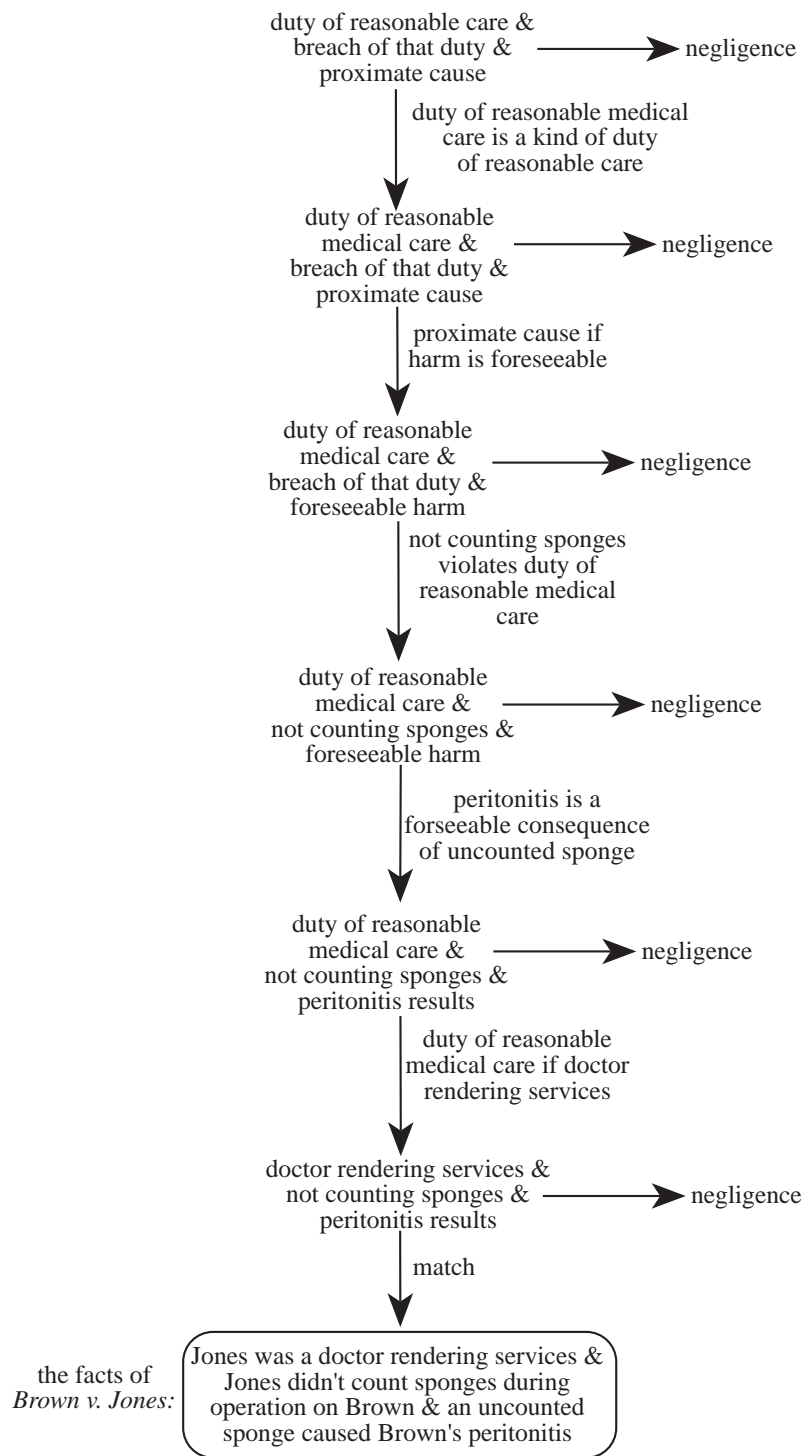sponge caused Brown's peritonitis

Figure 3: The justification for Jones' liability to Brown represented as a warrant-reduction graph.

reduction graph and (2) each reduction operation connecting successive warrants for negligence. Thus, the authoritative elements of the justification include not only the general rule "negligence follows from a duty of reasonable care and a breach of that duty that proximately causes harm," but also more specific warrants, including "negligence follows from a duty of reasonable medical care and a breach of that duty that proximately causes harm," and "not counting sponges violates the duty of reasonable medical care." If the Jones case were a precedent having the justification shown in figure 3, then it could legitimately be cited for any of these propositions, for each is necessary to the ultimate decision under this justification.

The reduction-graph model of *ratio decidendi* is therefore as follows: if the justification for the decision in a precedent is represented as a warrant-reduction graph, the authoritative elements of the justification include (1) each warrant for the ultimate result appearing in the reduction graph and (2) each warrant used as a reduction operator to connect successive warrants for the ultimate result. Warrants of both types meet Wambaugh's test: if the warrant were false, the decision would no longer follow from the material facts of the case and the remaining warrants. For example, if failing to count sponges were not a violation of reasonable medical care, then one of the elements of negligence would not be met. In terms of the goal-reduction graph shown in Figure 2, there would be a gap between the goal of showing that a "duty of reasonable medical care was violated" and the facts of *Brown v. Jones.*

4 Adequacy of the Reduction-Graph Model

The previous sections identified five characteristics of *ratio decidendi* described in jurisprudential literature and set forth a computational model of *ratio decidendi*, the reduction-graph model. This section argues that the reduction-graph accounts for the most important

of the characteristics identified in section two.

The first characteristic that an adequate model of *ratio decidendi* should account for is that the *ratio decidendi* of a case include the propositions of law, explicit or implicit in the opinion, that are necessary to the decision. As discussed above, the warrants constituting a reduction-graph representation of the justification for a decision each satisfy Wambaugh's criterion in that the negation of any such warrant would prevent the decision from following from the facts of the case and the remaining warrants in the justification. The reduction-graph model therefore provides a criterion for distinguishing *ratio decidendi* from *dictum* and accounts for the logical relationship among the legal propositions constituting the *ratio decidendi*. Thus, the reduction-graph model satisfies the first criterion (provided that the justification for a judicial decision is accurately characterized as consisting of a warrant for the ultimate issue in the case together with all inferences necessary to match the antecedents of the warrant to the facts).

The second characteristic of *ratio decidendi* is that a gradation of warrants for the ultimate result in the case can usually be found in a single precedent, as typified by *Donoghue v. Stevenson*. As illustrated in Figure 3, the warrant-reduction representation of a precedent's justification makes explicit the manner in which successive reduction steps give rise to warrants at a range of levels of abstraction. The reduction-graph model therefore accounts for the existence of multiple warrants at different levels of abstraction in a single case.

The third characteristic of *ratio decidendi* is that the *ratio decidendi* of a precedent must be grounded in the specific facts of the case. This criterion is satisfied because the lowest abstraction warrants in a reduction graph are exemplars, *i.e.,* warrants that express the connection between concrete case facts and abstract predicates. For example, the lowest abstraction warrants in the goal-reduction graph shown in Figure 2 (which are equivalent

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to the last three reduction steps in the warrant-reduction graph shown in figure 3) are (1) a duty of reasonably medical care arises when a doctor renders medical services (2) not counting sponges during an operation violates the duty of reasonable medical care, and (3) peritonitis is a foreseeable consequence of failing to count sponges during an operation. These warrants relate specific case facts (*e.g.,* failing to count sponges during an operation) to abstract legal predicates (*e.g.,* foreseeable harm).

Moreover, the antecedent of the lowest abstraction warrant in a warrant-reduction graph directly matches the material facts of the case. Thus, this warrant is an exemplar. Indeed, the lowest abstraction warrant in a warrant-reduction graph is very similar to Goodhart's model of *ratio decidendi*, consisting of the material facts of the case plus the decision justified by those facts.

Fourth, the reduction-graph model addresses the phenomenon that the precedential effect of a case depends not just on its material facts and the outcome justified by those facts, but also upon the theory under which the facts justified the outcome. This can be illustrated with Cross's example of *Bourhill v. Young.*

The two alternative justifications for the decision in *Bourhill v. Young* can be represented (in simplified form) by the warrant-reduction graphs shown in Figures 4 and 5.[12] Figure 4 represents the justification of *Bourhill* under the rationale that a driver owes no duty of care in respect of his driving to persons outside the area of reasonably foreseeable danger. At the top of Figure 4 is the rule that duty of reasonable care, breach of that duty, and harm proximately caused by the breach together imply negligence liability. The closed-world assumption (*i.e.,* the plaintiff's burden of proof) permits the converse of the contrapositive of this rule to be inferred, *i.e.,* if any of the conditions of the rule is false, there is no

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[12]These justifications are represented as warrant-reduction graphs, rather than by the equivalent goal-reduction graphs, because it is less convenient to represent the application of the closed-world assumption in a goal-reduction graph.
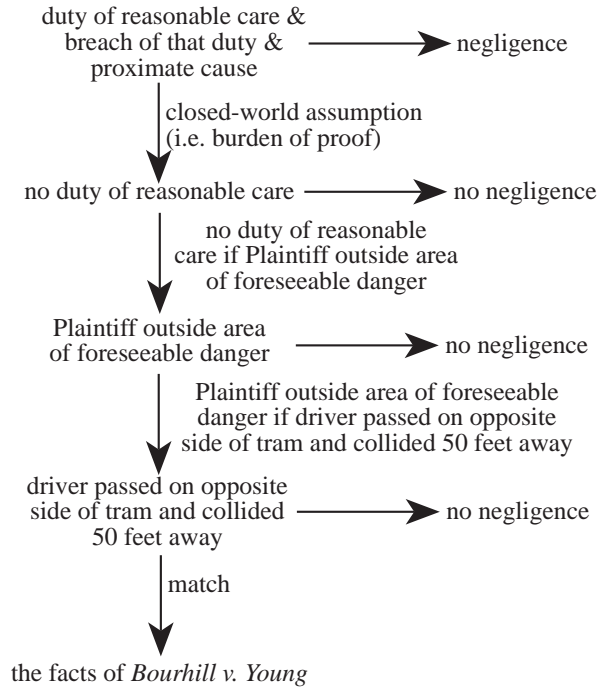
duty of reasonable care &
breach of that duty & ——————▶ negligence
proximate cause

closed-world assumption
(i.e. burden of proof)

no duty of reasonable care ——————▶ no negligence

no duty of reasonable
care if Plaintiff outside area
of foreseeable danger

Plaintiff outside area
of foreseeable danger ——————▶ no negligence

Plaintiff outside area of foreseeable
danger if driver passed on opposite
side of tram and collided 50 feet away

driver passed on opposite
side of tram and collided ——————▶ no negligence
50 feet away

match

the facts of *Bourhill v. Young*

Figure 4: The first theory of *Bourhill v. Young*.

negligence liability. The reduction operators necessary to match this rule to the facts of *Bourhill* under the first theory are the following:

- If a person is outside the area of foreseeable danger from an activity, then no duty of reasonable care is owed to that person by the actor.

- If a driver passes on the opposite side of a tram from which the plaintiff is alighting and has a collision 50 feet beyond the plaintiff, then the plaintiff is outside of the area of foreseeable danger.

Figure 5 represents the justification of *Bourhill* under the rationale that damages flowing from nervous shock are too remote a consequence of the breach of duty to be recoverable. The reduction operators necessary to match the rule under this theory are the following:

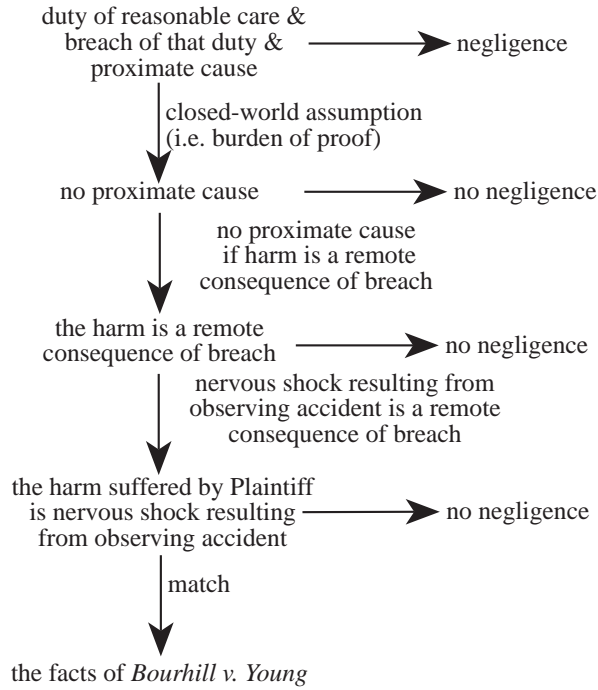- If the harm complained of is a remote consequence of the breach of duty of reasonable

Figure 5: The second theory of *Bourhill v. Young*.

care, then there is no proximate cause.

- Nervous shock suffered by an observer of an accident is a remote consequence of any breach of duty of reasonable care by the driver.

Recall the two hypothetical cases discussed above illustrating the inadequacy of material facts plus the decision thereon to represent *ratio decidendi*:

1. The same facts as *Bourhill* except that the motorcycle driven by Young collides with a fireworks truck instead of a car, and Mrs. Bourhill is burned by a fragment from the resultant explosion of fireworks.

2. The same facts as *Bourhill* except that Young passes the tram on the same side as the alighting Mrs. Bourhill, missing her by inches and causing a severe emotional shock.

Suppose that the first theory of *Bourhill* was intended by the House of Lords (as speeches

make clear was indeed the case). The lowest generality warrant in Figure 4 matches the facts of *H1* just as well as it matches as the facts of *Bourhill* itself. Thus, the justification for *Bourhill* under theory 1, which is represented in Figure 4, would apply equally to *H1*: Young would not be liable because Mrs. Bourhill was outside the area of foreseeable danger, notwithstanding that an unforeseeable causal chain led to her injury.

If the House of Lords had instead intended the second theory, the lowest generality warrant represented in Figure 5 would match the facts of *H2*. Thus, the justification for *Bourhill* under theory 2, which is represented in Figure 5, would apply equally to *H2*: nervous shock is a remote consequence of Young's breach of duty of reasonable care, notwithstanding that Mrs. Bourhill was within the area of foreseeable harm.

The antecedents of the lowest generality warrant in each reduction graph omit some facts that are arguably "material" to the cases. For example, the fact that Young was not exercising reasonable care in driving is surely a material fact, unlike, *e.g.*, Young's hair color. If the accident had been caused by a terrorist bomb exploding while Young was waiting at an intersection, it would have been unnecessary to address the issues of the scope of foreseeable danger or remoteness of Bourhill's injuries. However, Young's lack of reasonable care is not part of the *ratio decidendi* under either interpretation because it plays no role in the justification of the case under either interpretation, notwithstanding that there was in fact a determination that Young failed to exercise reasonable care. This is consistent with Wambaugh's criterion: the decision in the case would have been no different had the court believed that Young had exercised reasonable care in driving (although the case might have been decided on different grounds). The reduction-graph model therefore embodies a more restrictive criterion for relevance than Goodhart's criterion of materiality: only those facts appearing in the antecedents of a warrant in a precedent's justification are in the *ratio decidendi*.
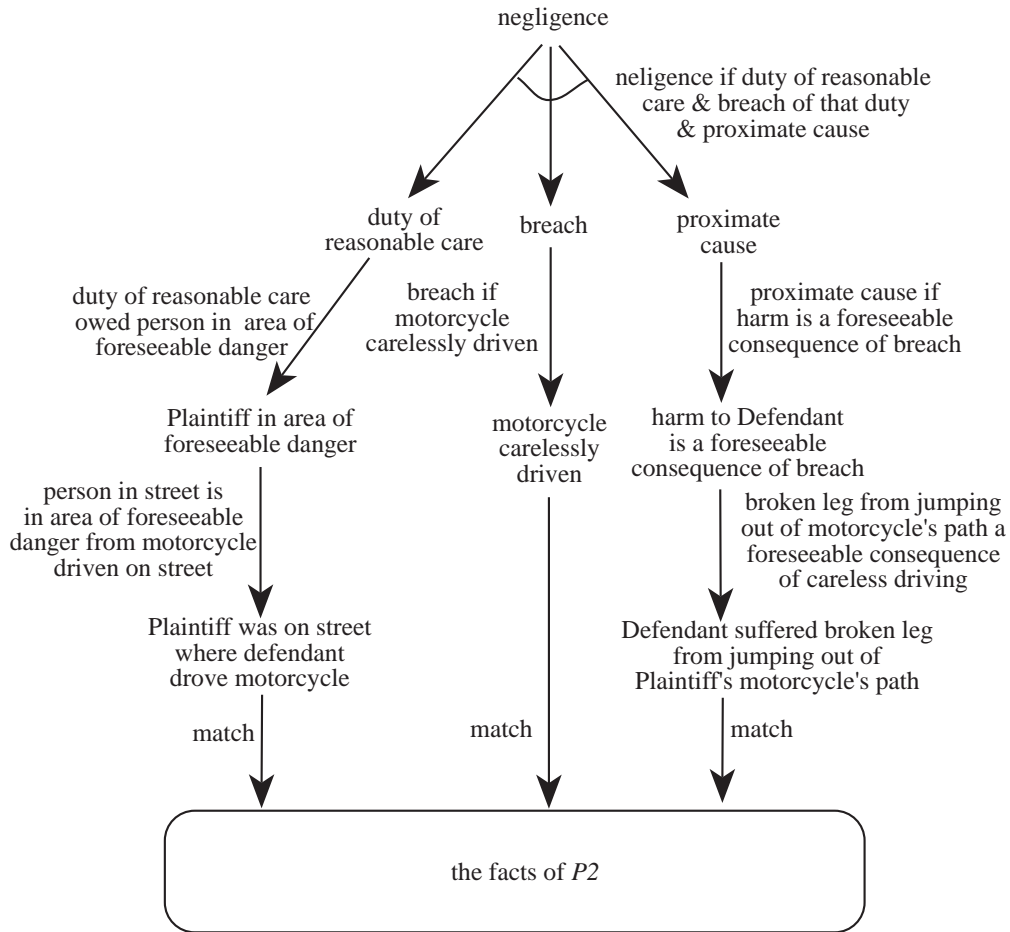
Figure 6: The justification for *P2* under theory 1.

Showing how negligence liability would follow in *H2* under theory 1 and in *H1* under theory 2 requires an additional precedent. For purposes of illustration, let *P2* be a precedent in which a carelessly driven motorcyclist passes on the same side as a passenger who has just alighted and is stepping across the street to the sidewalk. Jumping from the path of the motorcycle, the passenger trips and suffers a broken leg. The motorcyclist is held liable to the passenger for her injuries.

Figure 6 shows a justification for *P2* under theory 1 as a goal-reduction graph.[13] The

[13]The goal-reduction representation is used in Figures 6–9 because it is somewhat more compact than the equivalent warrant-reduction representation.
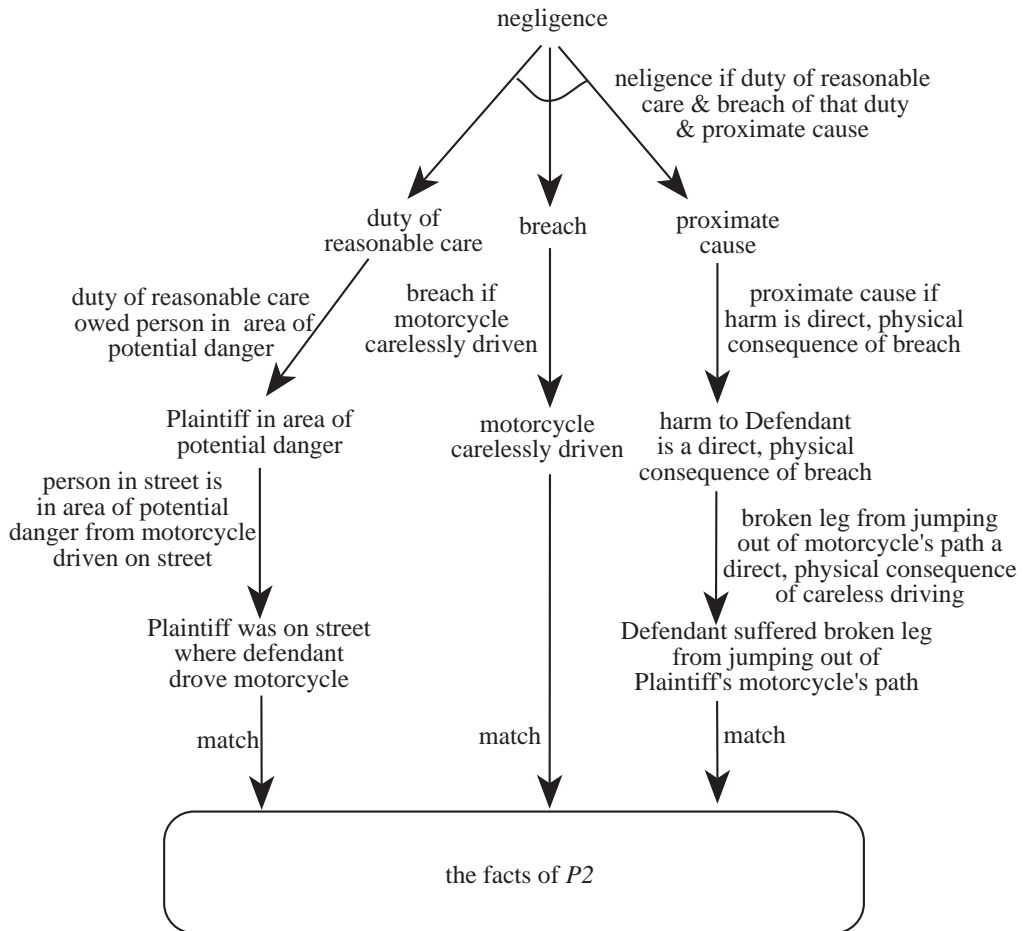
Figure 7: The justification for *P2* under theory 2.

reduction steps are:

- A duty of reasonable care is owed persons within the area of foreseeable danger from an activity.

- A person on a street is within the area of foreseeable danger from a motorcycle driven on the street.

- Carelessly driving a motorcycle is a breach of the duty of reasonable care.

- An action is the proximate cause of harm if the harm is a foreseeable consequence of the action.

- A broken leg suffered from when jumping from the path of a carelessly driven motorcycle is a foreseeable consequence of careless driving.

A justification for *P2* under theory 2 is shown in Figure 7. The reduction steps are:

- A duty of care is owed persons within the area of potential (though not necessarily foreseeable) danger from an activity.[14]

- A person on a street is within the area of potential danger from a motorcycle driven on the street.

- Carelessly driving a motorcycle is a breach of the duty of reasonable care.

- An action is the proximate cause of harm if the harm is a direct, physical consequence of the action.

- A broken leg suffered from when jumping from the path of a carelessly driven motorcycle is a direct, physical consequence of the carelessness.

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[14]This warrant was expressed by the dissent in *Palsgraf v. Long Island R.R. Co.*, 248 N.Y. 399, 162 N.E. 99 (1928).
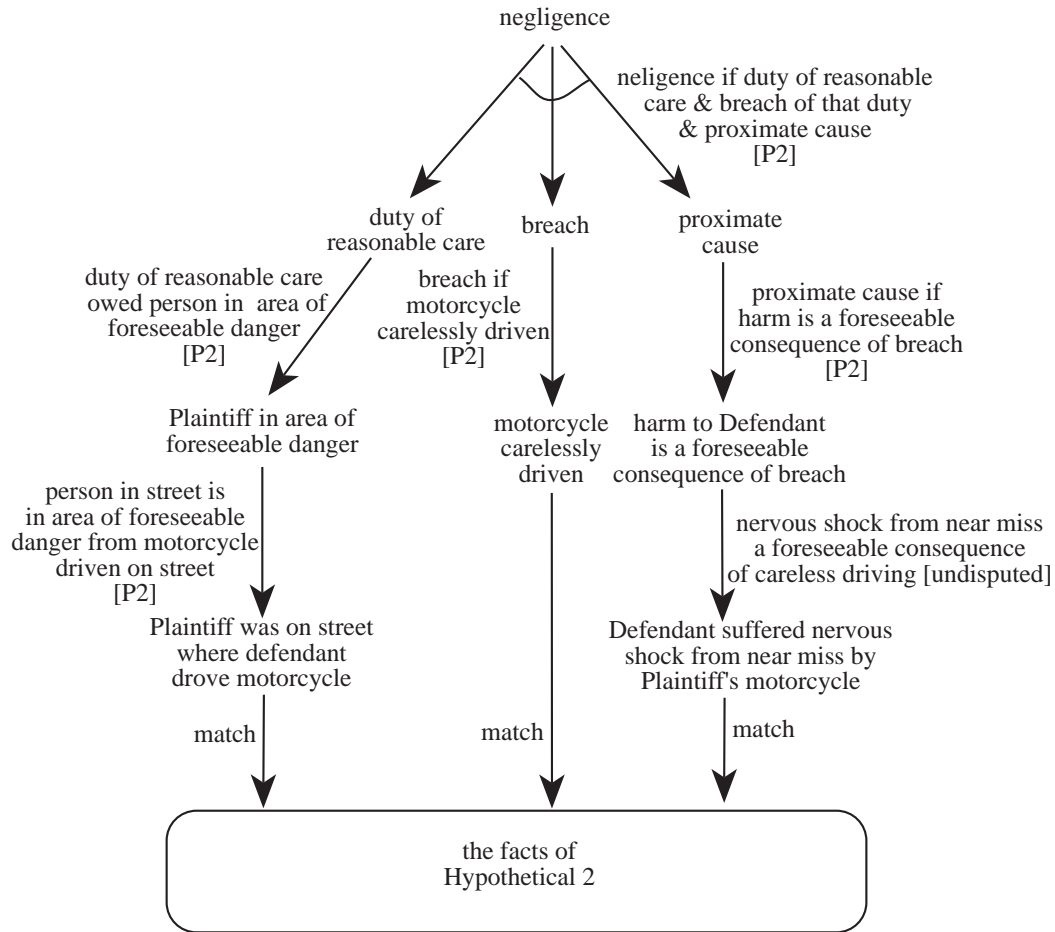
Figure 8: The justification for negligence in *H2* given that *P2* was decided under theory 1.

Assume that it is undisputed in *H1* that a burn resulting from the collision between the carelessly driven motorcycle and a fireworks truck is an unforeseeable but direct, physical consequence of the carelessness. Assume, in addition, that it is undisputed in *H2* that nervous shock from nearly being hit by a carelessly driven motorcycle is a foreseeable consequence of careless driving.

As with *Bourhill*, the precedential effect of *P2* depends on the particular theory under which it was decided. If *P2* were decided under theory 1, the justification for negligence liability shown in figure 8 could be constructed for *H2*. Warrants that are identical to those in *P2* are annotated with "[*P2*]", indicating that the *backing* [Toulmin, 1958]—that
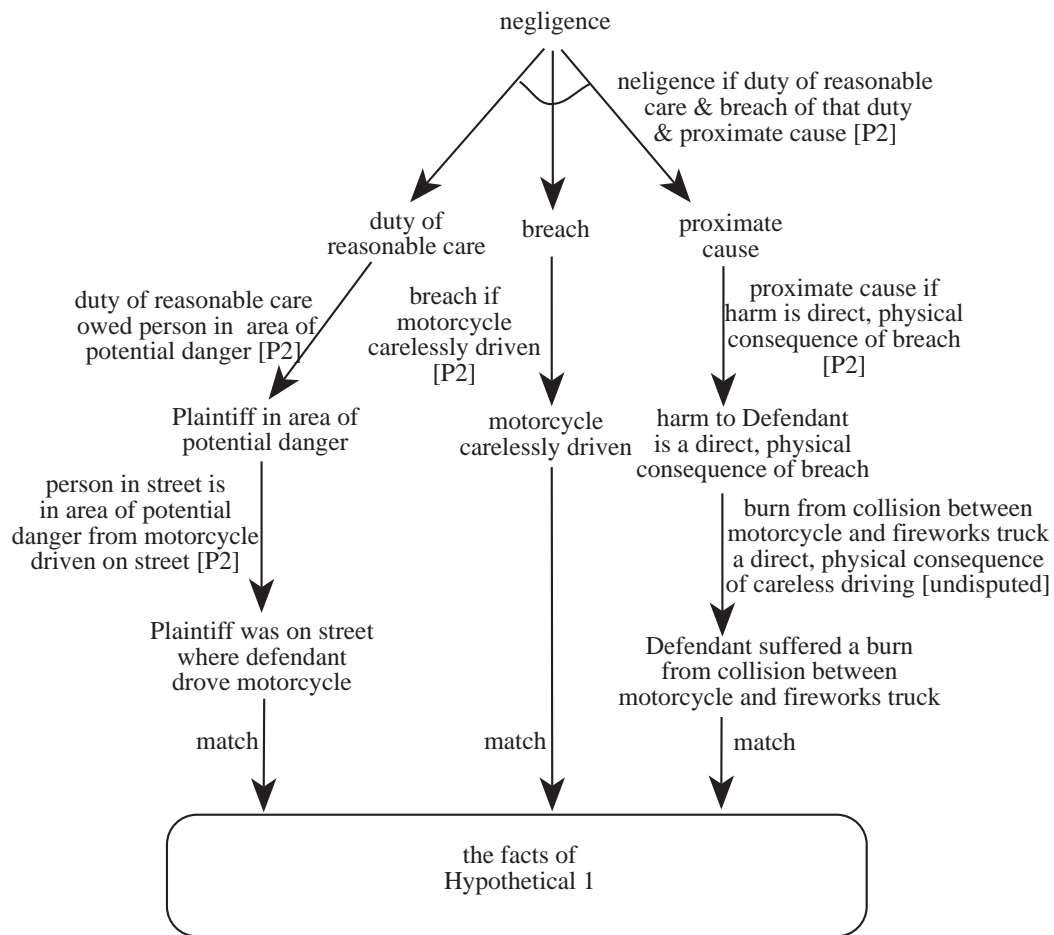
Figure 9: The justification for negligence in *H1* given that *P2* was decided under theory 2.

is, the reason for believing that the warrant's conclusion follows from its antecedent—is the authority of the court in *P2* to enunciate the warrant. The only warrant in this justification not present in *P2* is that nervous shock from nearly being hit by a carelessly driven motorcycle is a foreseeable consequence of the carelessness, which is undisputed. *P2* would not apply to *H1* because the injury in *H1* was not foreseeable, as required under this theory of decision.

If *P2* were instead decided under theory 2, the justification for negligence liability shown in Figure 9 could be constructed for *H1*. The only warrant in this justification not present in Figure 7 is that a burn resulting from a collision between the carelessly driven motorcycle and a fireworks truck is a direct, physical consequence of the carelessness, which is undisputed. *P2* would not apply to *H2* under this theory because nervous shock is not a physical consequence of the defendant's carelessness. Thus, the outcome in the hypotheticals under theory 2 is the reverse of the outcome under theory 1.

In summary, the reduction-graph model permits the theory of decision to control the case's precedential effect by explicitly representing the connection between the material facts of a case and the theory under which it was decided.

The fifth characteristic of *ratio decidendi* is that subsequent decisions can limit, extend, or overturn existing precedents. Two distinct forms of retroactive effect of precedents on the *ratio decidendi* of earlier precedents can be distinguished. One form of retroactive modification occurs when later cases incrementally extend or restrict the scope of an earlier precedent. Goodhart's reference to the metaphor that "the curve of law [requires] a series of points" is a recognition of the phenomenon that the precise scope of a precedent is determined in part by other precedents.

A second form of retroactive modification occurs when a later court explicitly adopts or rejects the holding of an earlier precedent or precedents. For example, the U.S. Supreme

Court in *Brown v. Board of Education*[15] explicitly rejected the "separate but equal" doctrine of *Plessy v. Ferguson*[16] as it applied to education.

Retroactive modifications of the first type arise from the nature of arguments based on exemplars. The persuasiveness of an argument based on the similarity between a new case and an exemplar of a predicate depends not just on the degree of relevant similarity between the new case and the exemplar, but also on the degree of relevant similarity between the new case and members of the predicate's *contrast set*, that is, exemplars of the predicate's negation. The scope of an exemplar therefore depends not only on the knowledge available for determining relevant similarity, but also on the propinquity of other exemplars. The boundaries of a legal concept in instance space, ill-defined when there are few exemplars, are gradually delineated as additional exemplars are successively added.[17]

Modeling this form of retroactive modification requires explicit representation of the exemplars corresponding to each precedent. The reduction-graph model satisfies this requirement, as do the exemplar-based based models discussed below in section five.

Two distinct aspects of retroactive modifications of the second type must be modeled: the effect of the adoption or rejection on the *ratio decidendi* of the earlier precedent; and the warrant in the later case expressing the adoption or rejection. Modeling the effect on the earlier case is trivial. If a warrant such as the "separate but equal" doctrine has been rejected, it should simply be removed from the knowledge base. No change need be made to a *ratio decidendi* that is adopted by in subsequent decision.

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[15]347 U.S. 483 (1954).

[16]163 U.S. 537 (1896).

[17]It has been noted that exemplar-based categories tend to be "hyper-polygons whose edges maintain equal 'distance' from nearest examples of different classes, where distance is measured by the system's similarity metric. These boundaries are somewhat analogous to contours of zero potential between positive and negative electric charges in physical space" [Clark, 1988].

Modeling the warrant in a later case expressing the adoption or rejection or an earlier warrant is somewhat more problematical. The reduction-graph representation includes all warrants necessary for the decision to follow from the material facts of the case under a particular rationale. The decision to reject one warrant in favor or another is, however, a meta-level decision that has no obvious place in the reduction graph *per se.* In effect, it represents instead a choice among possible reduction graphs rather than a step within a single reduction graph.

However, the rejection by a higher court of a given warrant, such as the "separate but equal" doctrine, is unquestionably binding on lower courts, notwithstanding that the rejection might not have been absolutely necessary for the decision in the higher court, *e.g.*, even if the earlier warrant could have been merely narrowed rather than outright rejected. This suggests that Wambaugh's criterion—that a proposition is part of the *ratio decidendi* of a decision only if the deciding court could not have believed the negation of the proposition without changing the outcome of the case—is too narrow to account for this aspect of *ratio decidendi.* The institutional obligation of courts to enunciate clear rules for the guidance of lower courts and potential litigants sometimes leads courts to explicitly alter prior decisions or mediate between competing lines of decision even when doing so is not strictly necessary for a decision in the case before the court. Because they are not strictly necessary for the outcome of the case, these explicit choices among rationales are not elements of a reduction graph, notwithstanding that they are binding on subsequent decisions.

In summary, the reduction-graph model can model the phenomenon that later cases incrementally extend or restrict the scope of earlier precedents because reduction graphs explicitly represent the exemplars in precedents' justifications. However, the reduction-graph model is not adequate to represent a court's choice among competing rationales.

5 Limitations of Purely Exemplar-Based Models of Ratio Decidendi

The previous sections have argued that exemplars are an essential component of the *ratio decidendi* of a case. This section argues, however, that exemplar-based reasoning is not *per se* sufficient as a model *ratio decidendi*. Specifically, a model of precedent consisting of representations of the material facts and outcome of precedents together with a single global measure of similarity is equivalent to Goodhart's model of precedent. Such a model is therefore subject to Cross's critique of Goodhart's models. The applicability of Cross's critique to purely exemplar-based models of *ratio decidendi* can be illustrated with the example *Bourhill v. Young.*

At least four different approaches to exemplar-based reasoning have been investigated. The first and simplest approach treats precedents as points in a feature space. The legal classification of a new case is determined by finding the new case's nearest neighbor in the feature space and applying the legal classification of that neighbor to the new case. The nearest neighbor is determined by a metric that typically consists of a weighted sum of featural differences, where the weight of a feature is intended to represent its relevance or "salience." A typical instance of this approach is PANNDA [Tyree, 1989]. This approach is a legal application of nearest-neighbor instance-based classification algorithms, which have been used for classification in a wide variety of domains. *See generally* [Aha, 1990].

A second approach also involves nearest-neighbor classification, but employs a structural representation of case facts and uses *structural similarity* between case facts as a similarity metric. Two cases are structurally similar if "objects in the cases can be placed into correspondence so that relations also correspond" [Holyoak and Thagard, 1989]. This approach, which was originally suggested as a model of precedential legal reasoning in

[Winston, 1980], was used in BRAMBLE [Bellairs, 1989].[18] The motivation for use of structural similarity as a similarity metric is the hypothesis that relevant similarity between cases is a function of "relational commonalities independent of the objects in which those relations are embedded." [Gentner, 1989].

A third approach, *dimensional analysis* [Ashley, 1990], uses knowledge of the factors that tend to establish or negate a predicate and the magnitude of these factors in precedents and new cases. These factors, or *dimensions*, provide criteria for determining the "most on-point" precedents and a mechanism for generating arguments based on a comparison between a new case and precedents. Specifically, a precedent is similar to a new case ("on-point") if the two cases share one or more dimensions. One precedent is more similar to a new case than a second precedent if the dimensions shared by the first precedent and the new case are a superset of those shared by the second precedent and the new case. The most similar ("most-on point") precedents are those that are similar and for which there is no more similar case. Arguments for a classification of a new case can be based on a comparison between a new case and the most similar precedents.

The final approach is the "prototype-plus-deformation" model used in TAXMAN II and described in [McCarty and Sridharan, 1982]. Under this model, a legal concept has three components. First, there is an optional invariant component that provides necessary, but not sufficient, conditions for satisfying the concept. Second is a set of exemplars providing sufficient conditions. Third, there is a set of transformations expressing various relationships among the exemplars. The result is an "ordered space of exemplars" in which a partial ordering is imposed on the exemplars by the transformations. Such a space can sometimes be represented by a single prototype together with a series of transformations, which act as "deformations." A legal argument for a particular classification is modeled

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[18]This approach was used for exemplar matching in GREBE [Branting, 1991a], but GREBE represented precedents as multiple exemplars.

as a set of transformations that includes all the exemplars and the given case. A counter-argument consists of a set of transformations that includes the exemplars but excludes the given case. The most persuasive argument is the one that "imposes the greatest degree of coherence on the set of exemplars."

Consider first the structural similarity approach. Whether *Bourhill* or *P2* has a higher degree of structural similarity to *H1* or to *H2* depends only on the representation used for the cases and not at all on the theory under which *Bourhill* and *P2* were decided. A representation that leads to the correct matches under one theory will necessarily lead to incorrect matches under the other theory. Therefore, the structural similarity approach is inadequate to distinguish among the possible *ratios* of *Bourhill*.

Under the dimensional approach, the precedents and hypotheticals might be represented as shown in Figure 10. There are two dimensions: foreseeability of harm, which ranges from none to high, and directness of consequences, which ranges from none to direct. Greater values along each of the dimensions tends to strengthen the plaintiff's claim for negligence. *Bourhill* and *P2* share all the same dimensions as *H1* and *H2* and are therefore a "most on point" cases.

The most that can be said about *H1* and *H2* under this approach is simply to compare the magnitudes of the two dimensions in the hypotheticals with those in *Bourhill* and *P2*. There is no way to represent either of the alternative theories under which either precedent has greater relevant similarity to one hypothetical than to the other.

A featural representation of the cases might be that each case consists of two features (foreseeability and directness) each with three possible values (the same as under the dimensional approach). The featural approach would differ from the dimensional approach in two respects. First, under the featural approach there would be no notion that changing a value feature in a particular direction would tend to establish or negate a given recovery
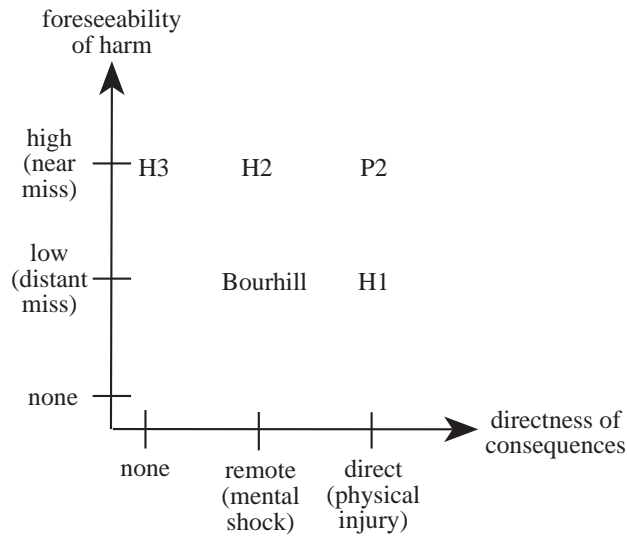
Figure 10: Featural and dimensional representations of precedents and hypotheticals.

by the claimant. Second, the featural approach would have explicit feature weighting.[19]

At first glance it appears that if such a weighting scheme could be devised, it would provide a mechanism for expressing the justification of individual precedents. For example, suppose that *Bourhill* and *P2* were decided under the first theory, under which *H1* should be controlled by *Bourhill* and *H2* should be controlled by *P2*. This could be modeled by placing a higher weight on the foreseeability feature than on the directness feature. If the distance function were $\sqrt{a|F_1 - F_2| + b|D_1 - D_2|}$, where F_n and D_n are the values (1–3) of foreseeability and directness of consequences respectively for case n, a and b are the weights of foreseeability and directness respectively, and adjacent feature values are separated by a unit distance, then setting $a > b$ would cause *H2*'s strongest match to be to *P2* and *H1*'s strongest match to be to *Bourhill*.

However, suppose that there is a third hypothetical, *H3*, in which the motorcyclist

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[19]The infeasibility of devising such a weighting scheme is discussed in [Ashley and Rissland, 1988].

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passes on the side of the tram from which the Plaintiff is alighting, missing her by inches. At that moment, an IRA bomb goes off, injuring Plaintiff. *H3*, shown in Figure 10, differs from *P2* by a distance of 2 in its directness feature but not at all in its foreseeability feature. It differs from *Bourhill* by a distance of 1 in each feature. Thus, *H3*'s difference from *P2* is $\sqrt{a|2-2|+b|0-2|} = \sqrt{2b}$, and *H3*'s difference from *Bourhill* is $\sqrt{a|2-1|+b|0-1|} = \sqrt{a+b}$. *H3* clearly should have the same result as *Bourhill*, since there is no direct connection between the motorcyclist's wrongdoing and the Plaintiff's injury. Thus, it must be the case that the distance from *P2*, $\sqrt{2b}$, is greater than the distance from *Bourhill*, $\sqrt{a+b}$. However, $\sqrt{2b} > \sqrt{a+b}$ implies that $b > a$, which contradicts the assumption that $a > b$. Thus, no plausible global feature weighting scheme can adequately model the effect of the justifications of *Bourhill* and *P2* on these hypotheticals.

Finally, the transformation coherence approach suffers from the same inadequacy as the other approaches. Whether the transformations from *Bourhill* to *H1* and *P2* to *H2* are more or less coherent than the transformations from *Bourhill* to *H2* and *P2* to *H1* depends only on the facts being transformed and not at all on the theory under which *Bourhill* and *P2* were decided.[20] Thus, this approach is also inadequate, standing alone, to model the the effects of different possible *ratios* of the precedents.

In summary, exemplar-based reasoning is essential for any adequate model of *ratio decidendi*, but a model of *ratio decidendi* that consists only of exemplar-based reasoning implicitly subscribes to Goodhart's view and is therefore subject to Cross's critique.

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[20]A recent description of the prototypes-and-deformations model suggests that it may now be intended to apply to the explanations, as well as the facts, of precedents [McCarty, 1991]. This refinement would represent a departure from the simple exemplar-based approach and therefore from the applicability of Cross's critique.

6 The Pragmatics of the Reduction-Graph Model

The reduction-graph model is a theory about the form of knowledge necessary to represent the *ratio decidendi* of legal precedents. Any implementation of this model must address several issues beyond the model itself, including the problem of exemplar matching, the role to be played by the backing of warrants, and phenomenon that judicial opinions typically fail to articulate all the reasoning steps necessary to justify the case decision.

6.1 The Problem of Exemplar Matching

Establishing a conclusion about a new case requires reduction operators to bridge the gap between abstract legal predicates and the specific facts of the new case. Exemplars play a central role in constructing justifications for conclusions about new cases because they connect specific facts to abstract features.

However, it is rare for the facts of any exemplar to precisely match the facts of a new case.[21] As a result, determining the abstract features satisfied by a new case typically requires determining the exemplar or exemplars having the highest degree of relevant similarity to the new case.[22] This determination requires a reliable criterion for degree of "relevant similarity" between pairs of cases.

Unfortunately, an appropriate criterion for relevant similarity between legal exemplars and new cases is difficult to devise. Typical formulations among legal scholars are that two cases share relevant similarity with respect to a legal category if the "principle of policy" [Bodenheimer, 1974], "rationale" [Murray, 1982], or "justification" [Raz, 1979] for membership in the category by the first applies as well to the second. An alternative

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[21]The hypotheticals posed in the context of *Bourhill* were contrived to minimize problems of exemplar matching.

[22]In the case-based reasoning literature, the task of determining the most relevant exemplar or exemplars is termed *indexing* or *retrieval* [Bareiss, 1991].

formulation by Fredrick Schauer focuses on the *category of assimilation* formed by equating two cases. Schauer views the task of a theory of precedent to be to explain how and why "in a world in which a single event may fit into many different categories ... some assimilations are plausible and others are not." [Schauer, 1987] at 581.

Precedents vary in the extent to which they articulate reasons that the facts of an exemplar satisfy a legal category. Often, precedents set forth no reasons why the facts of exemplars satisfy the legal categories of which they are members. Determining the degree of relevant similarity between the facts of an unjustified exemplar and a given case requires determining the most plausible justification that can be constructed to apply to both the exemplar and the new case. A number of sources of knowledge for constructing and evaluating such justifications have been identified, including social custom, historical development of legal doctrine, and models of social justice [Cardozo, 1921]. However, these knowledge sources are external to the precedents themselves. Often, these knowledge sources permit conflicting arguments to be constructed, but provide no firm ground for choosing among these arguments. A judge faced with such a case must engage in "interstitial" law making, imposing, rather than discovering or inferring, a justification common to the exemplar and the new case.

Sometimes a precedent sets forth a justification for an exemplar, but the justification is too general to be directly applicable. For example, knowing that the justification of an exemplar is that "no one should be permitted to profit from his own wrongdoing" may be of little help in resolving a subsequent case involving a lesser degree of wrongdoing, because the determinative issue—what degree of wrongdoing is sufficient to foreclose recovery—is not resolved by this general statement of policy. Just as under the first possibility, a more specific justification must be constructed and evaluated using knowledge of custom, history, and social justice. However, the possible justifications that such a precedent can share with

any other case are restricted under these circumstances to justifications consistent with this policy. Thus, a general statement of policy constrains, but doesn't obviate, the construction of new justifications.

Even if a precedent provides specific reasons why the facts of an exemplar satisfy a legal predicate, it may be problematical to determine whether these reasons apply strongly enough to a new case that the legal predicate should apply to the new case as well. The variety of complex factors that affect whether an entity satisfies a given category are explored in [Murphy and Medin, 1985] and [Lakoff, 1987].

Each of the four computational approaches discussed above in section five—nearest-neighbor, dimensional analysis, structure matching, and prototypes-and-deformations—accounts for certain aspects of the phenomenon of exemplar matching. However, none provides an complete model of the manner in which knowledge of social custom, historical development of legal doctrine, or models of social justice can guide decisions about the equivalence of cases with respect to a legal category.

Any implementation of the reduction-graph model must provide some mechanism for exemplar matching. However, the reduction-graph model itself is not tied to any position on the issue of which of the four existing computational methods best models the process of matching unexplained exemplars, or whether instead some alternative approach might be superior.

Regardless of the particular mechanism for exemplar matching or model of case similarity adopted, knowledge of the *ratio decidendi* of a precedent—that is, of a warrant for the ultimate issue in the case together with all reduction steps necessary to match the antecedents of the warrant to the facts—can simplify the task of exemplar matching by permitting the precedent to be viewed as consisting of multiple exemplars rather than as a single exemplar [Branting, 1991b]. Exemplar matching should therefore be applied to the

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smallest collections of case facts that justify legal conclusions in a precedent. Any coarser granularity matching compromises the ability to express and use the *ratio decidendi* of the precedent.

6.2   The Role of the Backing of Warrants

Under the reduction-graph model, the warrants in the justification of a judicial decision that are authoritative as to subsequent cases include a warrant for the ultimate result and all reduction operators necessary to match the antecedent of this warrant to the facts of the case. These warrants each satisfy Wambaugh's criterion because if any were negated the decision would no longer follow from the remaining warrants.

However, the warrants constituting the *ratio decidendi* of a precedent may themselves have justifications. The *backing* for a warrant consists of the reasons for believing that the warrant's consequent follows from its antecedents. The backing of a statutory rule consists of the authority under which the statute was enacted. Similarly, the backing for an existing common law rule consists of the authority of the court or courts enunciating the rule.

The nature of the backing for a new common law rule or exemplar is more complex. If an exemplar represents the resolution of a factual question concerning whether a given set of facts satisfies a particular legal predicate, then the backing for the exemplar at the appellate level is simply the institutional deference of the appellate court to findings by triers of fact. The rationale for new warrants representing the resolution of legal issues can include social custom, historical development, and models of social justice—the factors discussed above in the context of exemplar matching. However, the ultimate backing for such warrants is simply the institutional obligation of courts to resolve the disputes that come before them.

The rationale for a warrant that constitutes an element of the *ratio decidendi* of a case is

not itself part of the *ratio decidendi.* This is because the decision in a precedent would still follow from the warrants and facts of the case even if the rationale were absent or negated. In practice, subsequent courts may attempt to narrow or distinguish a warrant whose rationale is unsound or unpersuasive. However, the authoritative character of a precedent's warrants is not determined by the soundness of the rationale for those warrants. Law is at best only a partially deductive system. Judges must engage in interstitial law making because existing authoritative warrants are typically insufficient to deductively entail the resolution of new disputes.

The distinction between warrants and backing is illustrated by Goodhart's observation that many of the most influential precedents were decided on the basis of specious arguments. Among other examples, Goodhart discusses the case of *Hockster v. Delatour,*[23] in which it was held that suit for breach of contract could be brought after renunciation of the contract by the defendant but before the date on which the contract was to be performed. Lord Campbell, C.J., stated that "It is surely much more rational ... that, after the renunciation of the agreement by the defendant, the plaintiff should be at liberty to consider himself absolved from any future performance of it, retaining his right to sue." Goodhart quotes with approval Corbin's statement that "it does not follow therefrom that the plaintiff should be allowed to sue before the date fixed for performance by the defendant." Notwithstanding Campbell's *non sequitur, Hockster v. Delatour* became a leading case in contracts.

The rule that suit for breach of contract can be brought any time after renunciation is a warrant necessary for the decision in *Hockster v. Delatour, i.e.,* the decision in *Hockster* would no longer follow from facts of the case and the remaining warrants if this rule were absent or negated. Accordingly, the rule is authoritative as to subsequent cases. Lord

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[23]2 E. & B. 678 (1853).

Campbell's rationale for this rule, by contrast, is a nonessential part of the backing for the rule. Even in the absence of this rationale, the rule would nevertheless be valid because Lord Campbell was empowered to enunciate whatever propositions of law were necessary to resolve the dispute before him.

Even if Lord Campbell had ruled instead that a plaintiff must wait until the date on which a contract was to be performed before suing for breach, the rationale for this ruling would not have been a sound deduction, because the law of contract at the time of the decision in *Hockster v. Delatour* was insufficient to deductively resolve this issue.[24]

In summary, the authoritative character of warrants enunciated for the first time in a precedent comes from the institutional obligation of courts to resolve the disputes that come before them, even if doing so requires use of warrants that cannot be justified by sound deductions. Therefore, it is the warrants necessary for the resolution of a case and not the rationale for those warrants that are authoritative as to subsequent cases.

Although not part of the *ratio decidendi* of precedents, the purposes underlying decisions play an essential role in actual legal discourse [Berman and Hafner, 1993]. Accordingly, a complete theory of legal precedent must account for the teleological elements of precedents as well as their *ratio decidendi*.

6.3   Explanation Incompleteness

Even the most complete model of *ratio decidendi* is only useful to the extent that the *ratio decidendi* of precedents can in practice be determined. Unfortunately, judicial opinions seldom make explicit all inference steps necessary to justify a decision:

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[24]For example, a possible rationale for the opposite ruling would be that a contract can only be breached after the date on which the contract was to be performed. However, this reasoning simply begs the question whether renunciation of a contract itself constitutes a breach.

Incompleteness is the rule, not the exception; even a brief examination of cases makes this apparent ... Legal decisions contain obvious gaps .... [Warner, 1989] at 1532-1533.

Explanation indeterminacy is therefore a problem for any theory of *ratio decidendi*.

Richard Warner in *Three Theories of Legal Reasoning* [Warner, 1989] suggests that gaps in the reasoning of precedents can occur when there are several alternative rationales leading to the same conclusion. "[S]ince the court could decide the case without choosing any one alternative, the court may simply not have chosen any." Regardless of their source, such gaps can only be bridged by constructing a plausible inference step: "[C]ompleting incomplete legal reasoning is typically a matter of constructing premises, not of divining what unstated premises were in the mind of the court."

Three distinct approaches to the problem of explanation incompleteness in modeling *ratio decidendi* are possible. One approach is to adopt the most plausible completion of the missing reasoning steps. The result of this approach is a representation of the *ratio decidendi* under one construction of the decision.[25] This construction, and therefore the corresponding representation of the *ratio decidendi*, might have to be revised in light of subsequent decisions interpreting the precedent. In the meanwhile, however, a plausible and complete representation of the *ratio decidendi* would exist.

A second approach is to represent all plausible completions of the justification of a decision, suitably annotating the alternative reasoning steps. This amounts to representing a precedent as a set of plausible *rationes*. The benefit of this approach is that it provides a mechanism for modeling the ability of skillful attorneys to base a variety of plausible arguments on a single precedent by exploiting the ambiguities in the precedent's justification.

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[25]This approach was used in GREBE [Branting, 1991a].

The most conservative approach is to omit any implicit reasoning steps, retaining only those warrants stated explicitly. This approach has the benefit of precluding spurious warrants at the cost of making it impossible to model an attorney's ability to generate the plausible constructions of a precedent.

Some choice among the these approaches to explanation incompleteness must be made in any implemented model of *ratio decidendi*. Criteria for making this choice include the purposes for which the precedents are being modeled and the manner in which the representations of *rationes* will be used.

In summary, explanation incompleteness is a problem for any model of *ratio decidendi* and indeed for any model of precedent-based legal reasoning. The representation of the *ratio decidendi* of a precedent that best models attorneys' use of precedents is one in which "gaps" in the justification of the precedent have been bridged by one or more plausible constructions of the precedent.[26]

7 Conclusion

This paper has proposed a model of *ratio decidendi* under which the authoritative elements of a precedent include (1) each warrant for the ultimate result appearing in the warrant-reduction graph of the precedent's justification and (2) each reduction operator connecting successive warrants in this reduction graph. This collection comprises those warrants in the precedent's justification that satisfy Wambaugh's criterion: if any such warrant were false, then the outcome of the case would no longer follow from the remaining warrants. The reduction-graph model accounts for most of the important characteristics of *ratio decidendi* identified in the jurisprudential literature. In particular, the reduction-graph model shows

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[26]The task of automatically generating plausible justifications to bridge gaps in the reasoning of precedents is a form of *abduction* [Pople, 1973].

how the theory under which a case is decided controls its precedential effect. However, the reduction-graph model is not *per se* adequate to represent a choice among competing theories of decision, since this is equivalent to a choice among reduction graphs.

A purely exemplar-based model of precedent implicitly adheres to Goodhart's view of *ratio decidendi* and therefore suffer from its critical weakness: failure to represent the reasoning under which a decision follows from the material facts of the case. By contrast, the reduction-graph model explicitly represents this reasoning. This permits a new case to be matched against the smallest collections of case facts that justified a legal conclusion in a precedent rather than requiring a new case to match the entire facts of any one precedent.

A complete computational theory of legal precedent must address several issues beyond the scope of the reduction-graph model of *ratio decidendi*. One of the most critical is the nature of the processes of generating and evaluating plausible justifications for (1) exemplar matching and (2) bridging gaps in the reasoning of precedents. These processes depend on sources of knowledge external to precedents themselves, such as custom, historical development, models of social justice, and the purposes underlying legal warrants. An adequate model of plausible justification probably requires, in addition, a cognitive model of legal categories accounting for their formation, development, and variations in plausibility.

The effectiveness of any computational model of precedent is limited by the representational power of the case-description language in which the facts of precedents and new cases are expressed. This case-description language must be capable of expressing any legally significant distinctions among the facts of cases. There is a growing recognition that no representation less expressive than first-order predicate calculus is likely to be sufficient for this purpose [McCarty, 1989]. The development of more powerful and tractable representation techniques is critical to any useful computer model of precedent, and indeed to the success of the AI enterprise as a whole.

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Finally, a complete computational theory of legal precedent must specify a control strategy under which the warrants in the justifications of precedents are used to solve legal problems. Any such control strategy must at a minimum be capable of integrating warrants at various levels of abstraction. Various control strategies with this property have been implemented, including EXPANDER [Walker, 1992], CABARET [Skalak and Rissland, 1992], and GREBE [Branting and Porter, 1991]. Moreover, an adequate control strategy must be guided by a number of knowledge sources that have been neglected by current models of precedent, such as variations in precedential weight due to procedural posture or the prestige of the judge writing the opinion. *See generally* [Berman, 1991].

Regardless of the course of future developments in knowledge representation, cognitive models of legal concepts, and theories of plausible justification, any adequate computer implementation of precedent-base legal reasoning must go beyond the the traditional views of the nature of *ratio decidendi*. This paper has sought to demonstrate that any adequate model of *ratio decidendi* must include every warrant necessary for the justification of a precedent. No theory of *ratio decidendi* that omits any such warrant can adequately model the dependence of the precedential effect of a case upon both the specific facts of the case and the reasoning connecting those facts to the ultimate decision. By explicating the distinction between exemplars and precedents, and between exemplars and rules, the reduction-graph model provides a framework within which to address the key research issues of knowledge representation, cognitive models, and plausible justification.

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