A Reduction-Graph Model of Ratio Decidendi

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Abstract

This paper proposes a model of *ratio decidendi* as a justification structure consisting of a series of reasoning steps, some of which relate abstract predicates to other abstract predicates and some of which relate abstract predicates to specific facts. This model satisfies four adequacy criteria for *ratio decidendi* identified from the jurisprudential literature. In particular, the model shows how the theory under which a case is decided controls its precedential effect. By contrast, a purely case-based model of *ratio* fails to account for the dependency of precedential effect on the theory of decision.

1 Introduction

Every computational model of legal precedent that refers to individual past cases necessarily embodies, at least implicitly, some model of *ratio decidendi*, the content of a precedent that is authoritative as to subsequent cases. Predicting, advocating, and justifying the binding effect of a precedent on subsequent cases all require identifying the authoritative elements of the precedent and applying those elements to subsequent cases.

This paper proposes a model of *ratio decidendi*, termed the *reduction graph* model, under which the *ratio decidendi* of a precedent is a justification structure consisting of a series of reasoning steps, some of which relate abstract predicates to other abstract predicates and some of which relate abstract predicates to specific facts. The next section argues that the jurisprudential literature on legal precedent provides a set of criteria for the adequacy of models of *ratio decidendi*. Section three describes the reduction-graph model and argues that it satisfies these criteria. The fourth section shows that a purely case-based model of legal precedent consisting of representations of the material facts of each precedent case together with a global relevance metric fails to satisfy an important criterion: representing how the theory under which a case is decided controls its precedential effect.

The reduction-graph model is intended as a *knowledge level* [New82] description of precedent-based reasoning, that is, a "specification of what a reasoning system should be able to do" independent of any particular "symbol-level" implementation of this process. The emphasis of this paper is therefore on identifying the knowledge required for precedent-based reasoning and showing how this knowledge is used in the resolution of new cases.

2 Evaluation Criteria for Models of *Ratio Decidendi*

Development of a satisfactory computer model entails three distinct tasks. First, the phenomenon to be modeled must be precisely described. Second, a set of appropriate computational mechanisms must be defined. Finally, the proposed computational mechanisms must be evaluated in terms of their ability to account for the phenomenon.

Several knowledge sources for describing the phenomenon of precedent-based legal reasoning are possible. The most reliable source of information would be detailed empirical studies of judges' and attorneys' use of precedents in problem solving. Unfortunately, few such empirical studies exist. One alternative is introspection on one's own use of precedents, but introspection is notoriously unreliable [Gar85]. However, the law's "tradition of examining its processes and assumptions" [Ris90] as embodied in jurisprudential literature provides another alternative. While jurisprudential writings are often contradictory and frequently serve a prescriptive rather than a descriptive agenda, jurisprudential ideas that survive prolonged critical scrutiny are likely to contain an element of truth.

Four widely recognized characteristics of *ratio decidendi* can be distinguished:

1. The *ratio* of a precedent consists of propositions of law, explicitly or implicitly set forth in the opinion, that are necessary to the decision. This characterization is supported by two observations. The first is that judicial decisions almost invariably have written justifications containing propositions of law that judges purport to apply to resolve the issues before them. The second is that not every part of a written justification is authoritative. As early as 1673,[1] it was recognized that written decisions frequently contain language unnecessary for the resolution of the issues before the court and that this unnecessary language is not part of the *ratio decidendi* of the case.

Predictability, judicial economy, and the principle that like cases should be treated alike all suggest that the legal propositions required resolve the issues in one case should apply equally to relevantly similar future cases. A widely accepted test for determining whether a given proposition is the *ratio* of a precedent was proposed by Eugene Wambaugh: if the deciding court could have believed the negation of the proposition without changing the outcome of the case, the proposition is *dictum* rather than *ratio* [Wam94].

2. A unique proposition of law "without which the case must have been decided otherwise" [Cro79] can seldom be determined. Instead, a gradation of propositions ranging in abstraction from the specific facts of the case to abstract rules can satisfy this condition. Rupert Cross in "Precedent in English Law" [Cro79] illustrated this point with the example of *Donoghue v. Stevenson*,[2] a case holding the manufacturer of a bottle of ginger beer containing a decomposed snail liable to the ultimate consumer. Lord Atkin's opinion contained two propositions of law justifying the decision. The first was very general:

> A party must take reasonable care to avoid acts or omissions which he can reasonably foresee would be likely to injure persons ...closely and directly affected by his act ....

The second was much more specific:

> A manufacturer of products, which he sells in such a form as to show that he intends them to reach the ultimate consumer in the form in which they left him with no reasonably possibility of intermediate examination ...owes a duty to the consumer to take ...reasonable care.

Cross observes that both of these propositions satisfy Wambaugh's test: the negation of either proposition would require a different decision.

3. The *ratio decidendi* of a precedent must be grounded in the specific facts of the case. The primary source of uncertainty in legal reasoning is the "gap" in generality between abstract legal concepts and the specific facts of new cases [Gar87, BP91]. Precedents are useful in deciding new cases because they provide examples of specific facts that satisfy such concepts. Accordingly, a model of *ratio* that omits specific case facts cannot fully account for the precedential effect of past cases.

Typical expressions of the centrality of precedents' specific facts include:

- "[I]t is the facts, and not the general rules of law found in precedents ... which serve as the foundation of the decisional process." [CR81] at 56.

- "Judgements must be read in light of the facts of the case in which they are delivered." [Cro79] at 44.

- "It is clear that the most important part of a representation for cases is the representation of the facts and the outcome." [Gar87] at 47. Gardner quotes with approval Corbin's admonition that cases should be studied "not so much for their doctrinal statements as for ...their 'operative facts' " and Gilmore's statement that one should "never quote general language from an opinion, divorced from the factual context in which the language was delivered."

[1] *Bole v. Horton*, Vaughn 360.

[2] A.C. 562 (1932).

4. The *ratio decidendi* of a precedent includes not only the precedent's material facts and decision, but also the theory under which the material facts lead to the decision. Although numerous legal scholars have emphasized that specific facts are essential to any model of *ratio decidendi*, specific facts are not *per se* sufficient to represent *ratio*. A consensus on this conclusion emerged over the decades following Arthur Goodhart's publication of a model of *ratio* that focused on the facts of precedents to the exclusion of the judge's purported justifications. Goodhart argued that "the *ratio decidendi* of a case ... must not be sought in the reasons on which the judge has based his decision" but must be sought instead in "the material facts as seen by the judge and his conclusion based on them. It is by his choice of the material facts that the judge creates law" [Goo30].

The most important criticism of Goodhart's view is that:

> it is quite impossible to formulate the *ratio decidendi* merely by reference to the facts, regarded as material by the court, and the decision based on those facts. It is often essential to know why certain facts were regarded as material and for this purpose it may be necessary to know what portions of the law were in the mind of the court when the selection was made. [Cro79] at 73.

Cross gives the example of *Bourhill v. Young,*[3] in which the following facts were found to be material:

> Young, a motorcyclist, was killed because of his own negligence when he passed a tram at excessive speed and collided with a car about 50 feet beyond the tram. At the time of the accident, the tram was stopped and Mrs. Bourhill was alighting. Mrs. Bourhill heard the collision and saw blood on the road after the accident and as a result suffered a nervous shock. Mrs. Bourhill was outside what Young ought to have contemplated as the area of potential danger that would arise from his careless driving.

[3] A.C. 92 (1943).

The decision was that Mrs. Bourhill's action against Young's estate was dismissed.

From the material facts and the decision alone it is impossible to determine which of the following two rationales underlies the decision: (1) a driver owes no duty of care in respect of his driving to persons outside the area of reasonably foreseeable danger, or (2) although the driver owes a duty of care to such persons, damages flowing from nervous shock are too remote a consequence of the breach of duty to be recoverable.

Without knowledge of the controlling rationale, it is impossible to determine from the material facts and the decision alone how either of the following hypotheticals should be decided:

- *H1.* The same facts as *Bourhill* except that the motorcycle driven by Young collides with a fireworks truck instead of a car, and the resultant explosion of fireworks causes the branch of a tree damaged by an early winter storm to come loose and hit Mrs. Bourhill on the head, resulting in physical injuries.
- *H2.* The same facts as *Bourhill* except that Young passes the tram on the same side as the alighting Mrs. Bourhill, missing her by inches and causing a severe emotional shock.

Under the first rationale, Mrs. Bourhill could recover in hypothetical 2 but not hypothetical 1. Under the second rationale, the results would be reversed.

An adequate model of precedent should, at a minimum, account for these four recognized characteristics of *ratio decidendi*. Accordingly, these characteristics can function as evaluation criteria for models of precedent-based reasoning.[4]

3 The Reduction Graph Model of *Ratio Decidendi*

Legal warrants, *i.e.,* propositions expressing the conditions under which a legal predicate is satisfied, vary widely in abstraction [BP91]. At one

[4] The four characteristics described here constitute only a partial description of *ratio decidendi*. A discussion of additional attributes of *ratio* together with an analysis of the adequacy of the reduction-graph model in terms of these additional attributes is set forth in [Bra93].

extreme are general rules, which express the requirements for a legal predicate in terms of other abstract predicates (*e.g.,* an action is negligent if the defendant fails to use reasonable care and the failure is the proximate cause of harm suffered by the plaintiff). At the other extreme are *exemplars*, collections of facts, expressed in a concrete case-description language, that are known to satisfy a legal predicate (*e.g.*, Dr. Jones was liable for negligence because he failed to exercise reasonable medical care by counting sponges during surgery on Brown. As a result, a sponge was left in Brown, who developed peritonitis and required a second operation to remove the sponge).[5]

The relationship between warrants at different levels of generality is expressed by *reduction operators*. Each reduction operator expresses a taxonomic relationship between the antecedents of different warrants for the same conclusion (*e.g.,* breach of the standard of reasonable care of a medical community is a kind of failure to use reasonable care). Reduction operators are themselves warrants, since they express the connection between an abstract feature and a set of conditions under which the abstract feature is satisfied. In particular, reduction operators that justify conclusions in terms of facts expressed in the case-description language (*e.g.*, Dr. Jones' failure to count sponges was a failure to conform to the standard of reasonable care of a medical community) are exemplars.

Establishing a conclusion about a new case requires matching the facts of the new case with the conditions of some authoritative warrant for that conclusion. However, matching the antecedents of a warrant to the facts of a new case typically requires inference. Warrants other than exemplars contain abstract features (*e.g.*, reasonable care) in their antecedents. Matching these abstract features to the specific facts of a new case (*e.g.*, failure to count the sponges used during surgery) requires reduction operators to bridge the gap between the abstract features and the specific facts (*e.g.*, the reduction operator that identifies failure to count the sponges used during surgery as a kind of failure to exercise reasonable care). Similarly, matching the facts of an exemplar to the facts of a new case may require inference to establish the equivalence of different facts (*e.g.*, failing to monitor blood oxygen level is similar to failing to count the sponges used during surgery because both are failures to conform to the standard of reasonable care of the medical community).

[5] In an unfortunate proliferation of terminology, this author has previously referred to exemplars as "exemplar-based explanations" and "precedent constituents" [BP91].

Figure 1: The justification, represented as a goal-reduction graph, for Jones' liability to Brown under negligence for the injuries that resulted from a sponge left by Jones in Brown's abdominal cavity during an operation.

A justification for the conclusion that a predicate applies to a case therefore consists of a warrant for the predicate together with all reductions necessary to match the antecedents of the warrant to the facts of the case. Various representations of such a justification are possible. Figure 1 represents a justification in terms of the subgoals that arise in the process of constructing an inference path from the predicate to be established to the facts of a case. The predicate to be established, negligence liability, is at the top. The rule that duty of reasonable care, breach of that duty, and proximately caused harm imply negligence permits this goal to be reduced to the subgoals of establishing a duty of reasonable care, breach of that duty, and proximately caused harm. These subgoals can in turn be reduced by reduction operators to the facts of the case: that Jones was a doctor rendering professional services in operating on Brown, that Jones failed to count sponges during the operation, and that Brown developed peritonitis from the sponge left during the operation. This representation is termed a *goal-reduction graph.*

Figure 2: The justification for Jones' liability to Brown represented as a reduction graph.

Figure 2 shows an equivalent alternative representation of this justification in which reduction operators are used to repeatedly rewrite the antecedents of a warrant for negligence until it matches the facts of the case. This representation is referred to as a *warrant-reduction graph.* Goal-reduction graphs and warrant-reduction graphs are simply alternative representations of a common underlying reduction graph [Ama68]. The two representations differ only in that the order of reduction operator application and the resulting warrants for the ultimate result are made explicit in a warrant-reduction graph, whereas the subgoals addressed by each reduction operator are made explicit in the goal-reduction graph.

3.1 The Elements of *Ratio Decidendi*

A judicial decision consists of a determination that some legal predicate, *e.g.*, negligence liability, is satisfied by the facts of a case as determined by the trier of fact. If the decision has a justification in terms of these facts, this justification must necessarily include a warrant for the predicate together with all inferences necessary to match the antecedents of the warrant to the facts. Predictability, judicial economy, and the principle that like cases should be treated alike all argue that each warrant in this justification should apply equally to similar future cases.

In figure 2, for example, the warrants necessary for the ultimate conclusion that negligence liability applies to Jones include (1) each warrant for negligence appearing in the reduction graph and (2) each reduction operation connecting successive warrants for negligence. Thus, the authoritative elements of the justification include not only the general rule "negligence follows from a duty of reasonable care and a breach of that duty that proximately causes harm," but also more specific warrants, including "negligence follows from a duty of reasonable medical care and a breach of that duty that proximately causes harm," and "not counting sponges violates the duty of reasonable medical care." If the Jones case were a precedent having the justification shown in Figure 2, then it could legitimately be cited for any of these propositions, for each is necessary to the ultimate decision under this justification.

The reduction-graph model of *ratio decidendi* is therefore as follows: if the justification of the decision in a precedent is represented as a warrant-reduction graph, the authoritative elements of the justification include (1) each warrant for the ultimate result appearing in the reduction graph and (2) each warrant used as a reduction operator to connect successive warrants for the ultimate result. Warrants of both types meet Wambaugh's test: if the warrant were false, then (at least in the absence of some alternative justification) the outcome of the case would necessarily be otherwise. For example, if failing to count sponges were not a violation of reasonable medical care, then one of the elements of negligence would not be met. In terms of the goal-reduction graph shown in Figure 1, there would be a gap between the goal of showing that a "duty of reasonable medical care was violated" and the facts of Brown v. Jones.

3.2 Adequacy of the Reduction-Graph Model

This subsection argues informally that the reduction-graph model satisfies the criteria set forth in section 2. The first characteristic that an adequate model of *ratio decidendi* should account for is that the *ratio* of a case include the propositions of law, explicit or implicit in the opinion, that are necessary to the decision. As discussed above, the warrants constituting a reduction-graph representation of the justification for a decision each satisfy Wambaugh's criterion in that the negation of any such warrant would prevent the decision from following from the facts of the case. The reduction-graph model therefore distinguishes necessary from unnecessary propositions, *i.e.*, *ratio* from *dictum*, and accounts for the logical relationship among the legal propositions constituting the *ratio*. Thus, the reduction-graph model satisfies the first criterion (provided that the justification for a judicial decision is accurately characterized as consisting of a warrant for the ultimate issue in the case together with all inferences necessary to match the antecedents of the warrant to the facts).

The second characteristic of *ratio decidendi* is that a gradation of warrants for the ultimate result in the case can usually be found in a single precedent. The warrant-reduction representation of a precedent's justification makes explicit the way in which successive reduction steps give rise to warrants at a range of levels of abstraction, as illustrated in Figure 2.

The third characteristic is that the *ratio* of a precedent must be grounded in the specific facts of the case. This criterion is satisfied because the lowest abstraction warrants in a reduction graph are exemplars, *i.e.,* warrants that express the connection between concrete case facts and abstract predicates. For example, the lowest abstraction warrants in the goal-reduction graph shown in Figure 1 are (1) a duty of reasonably medical care arises when a doctor renders medical services (2) not counting sponges during an operation violates the duty of reasonable medical care, and (3) peritonitis is a foreseeable consequence of failing to count sponges during an operation. These warrants relate specific case facts (*e.g.,* failing to count sponges during an operation) to abstract legal predicates (*e.g.,* foreseeable harm).

Finally, the reduction-graph model also addresses the phenomenon that the precedential effect of a case depends not just on its material facts and the outcome justified by those facts, but also upon the theory under which the facts justified

Figure 3: The first theory of *Bourhill v. Young.*

the outcome. This can be illustrated with Cross's example of *Bourhill v. Young.*

The two alternative justifications for the decision in *Bourhill v. Young* can be represented (in simplified form) as shown in Figures 3 and 4. Figure 3 represents the justification of *Bourhill* under the rationale that a driver owes no duty of care in respect of his driving to persons outside the area of reasonably foreseeable danger. At the top of Figure 3 is the rule that duty of reasonable care, breach of that duty, and harm proximately caused by the breach together imply negligence liability. The closed-world assumption (*i.e.,* the plaintiff's burden of proof) permits the converse of the contrapositive of this rule to be inferred, *i.e.*, if any of the conditions of the rule is false, there is no negligence liability. The reduction operators necessary to match this rule to the facts of *Bourhill* under the first theory are the following:

- If a person is outside the area of foreseeable danger from an activity, then no duty of reasonable care is owed to that person by the actor.

- If a driver passes on the opposite side of a tram from which the plaintiff is alighting and has a collision 50 feet beyond the plaintiff, then the plaintiff is outside of the area of foreseeable danger.

Figure 4: The second theory of *Bourhill v. Young.*

Figure 4 represents the justification of *Bourhill* under the rationale that damages flowing from nervous shock are too remote a consequence of the breach of duty to be recoverable. The reduction operators necessary to match the rule under this theory are the following:

- If the harm complained of is a remote consequence of the breach of duty of reasonable care, then there is no proximate cause.
- Nervous shock suffered by an observer of an accident is a remote consequence of any breach of duty of reasonable care by the driver.

Suppose that the first theory of *Bourhill* was intended by the House of Lords (as speeches make clear was indeed the case). The lowest generality warrant in Figure 3 matches the facts of the first hypothetical just as well as it matches as the facts of *Bourhill* itself. Thus, the justification for *Bourhill* under theory 1, which is represented in Figure 3, would apply equally to the first hypothetical: Young would not be liable because Mrs. Bourhill was outside the area of foreseeable danger, notwithstanding that an unforeseeable causal chain led to her injury.

If the House of Lords had instead intended the second theory, the lowest generality warrant represented in Figure 4 would match the facts of the second hypothetical as well as it matches the facts of *Bourhill* itself. Thus, the justification for *Bourhill* under theory 2, which is represented in Figure 4, would apply equally to the first hypothetical: nervous shock is a remote consequence of Young's breach of duty of reasonable care, notwithstanding that Mrs. Bourhill was within the area of foreseeable harm.[6]

The reduction-graph model, by explicitly representing the connection between the material facts of a case and the theory under which it was decided, permits the theory of decision to control the case's precedential effect. This improves case matching by permitting precedents to be viewed as multiple exemplars rather than as a single exemplar [BP91] and improves the quality of precedent-based explanations by framing these explanations in terms of the specific issues addressed by the court in its decision.

3.3 Implementation of the Reduction-Graph Model

The reduction-graph model is a theory about the form of knowledge necessary to represent the of *ratio decidendi* of legal precedents. A number of additional details must be specified by any particular implementation of the model. The most critical element is a case-description language in which to express the facts of the exemplars contained in the em ratio of each precedent. This case-description language must be capable of expressing any legally significant distinctions among the facts of cases. There is a growing recognition that no representation less expressive than first-order predicate calculus is likely to be sufficient for this purpose [McC89].

A second requirement is a mechanism for exemplar-based reasoning. The hypotheticals posed in the context of *Bourhill* were contrived to minimize problems of case matching. In reality, few pairs of distinct cases can be found with identical material facts. As a result, justifying a conclusion with an exemplar almost invariably requires reasoning about factual similarities and differences between

[6]Space limitations preclude showing how negligence liability would follow in hypothetical 2 under theory 1 and in hypothetical 1 under theory 2 if there were an additional precedent, *P2*, in which a motocyclist passing on the same side as an alighting passenger and striking the passenger was held to be liable for negligence under the same theory as applies in *Bourhill.* See [Bra93] for a more complete discussion.

a new case and the facts of the exemplar. While knowledge of a precedent's *ratio* can reduce the problem of exemplar matching by distinguishing multiple exemplars within a single precedent, it does not eliminate this problem.

At least four different approaches to exemplar-based reasoning have been investigated. The first and simplest approach treats precedents as points in a feature space. The legal classification of a new case is determined by finding the new case's nearest neighbor in the feature space and applying the legal classification of that neighbor to the new case. The nearest neighbor is determined by a metric that typically consists of a weighted sum of featural differences, where the weight of a feature is intended to represent its relevance or "salience." A second approach, *structure matching*, also involves nearest-neighbor classification, but employs a structural representation of cases and uses *structural similarity* as a similarity metric. Two cases are structurally similar if "objects in the cases can be placed into correspondence so that relations also correspond" [HT89].

A third approach, *dimensional analysis* [Ash88], uses knowledge of the factors that tend to establish or negate a predicate and the magnitude of these factors in precedents and new cases. These factors, or *dimensions*, provide criteria for determining the "most on-point" precedents and a mechanism for generating arguments based on a comparison between a new case and precedents. The final approach is the "prototype-plus-deformation" model used in TAXMAN II and described in [MS82]. Under this model, a legal argument for a particular classification is modeled as a set of transformations that includes all the exemplars of the classification and the given case. A counter-argument consists of a set of transformations that includes the exemplars but excludes the given case. The most persuasive argument is the one that "imposes the greatest degree of coherence on the set of exemplars." One of these four approaches must be selected as the exemplar-based reasoning component of any implementation of the reduction-graph model.

Finally, an implementation of the model must include a control strategy capable of integrating warrants of varying degrees of abstraction into a single solution. Various control strategies with this property have been implemented, including EXPANDER [Wal92], CABARET [SR92], and GREBE [BP91].

4 Limitations of a Purely Case-Based Model of *Ratio*

The previous sections have argued that exemplars are an essential component of the *ratio* of a case. This section argues, however, that exemplar-based reasoning is not *per se* sufficient as a model *ratio*. Specifically, a model of precedent consisting of representations of the material facts and outcome of precedents together with a single global measure of similarity is equivalent to Goodhart's model of precedent. Such a model is therefore subject to Cross's critique of Goodhart's models. This can be illustrated with the example set forth in the previous section.

Consider first the structural similarity approach. Whether *Bourhill* has a higher degree of structural similarity to *H1* or to *H2* depends only on the representation used for the cases and not at all on the theory under which *Bourhill* was decided. A representation that leads to the correct matches under one theory will necessarily lead to incorrect matches under the other theory. Therefore, the structural similarity approach is inadequate to distinguish among the possible *ratios* of *Bourhill*.

Under the dimensional approach, *Bourhill* and hypotheticals might be represented as shown in Figure 5. There are two dimensions: foreseeability of harm, which ranges from none to high, and directness of consequences, which ranges from none to direct. Greater values along each of the dimensions tends to strengthen the plaintiff's claim for negligence. Let *P2* represent the precedent of liability mentioned above (a motorcyclist passing on the same side as an alighting passenger and striking the passenger is liable for negligence under the same theory as applies in *Bourhill*). *Bourhill* and *P2* share all the same dimensions as *H1* and *H2* and are therefore a "most on point" cases.

The most that can be said about *H1* and *H2* under this approach is simply to compare the magnitudes of the two dimensions in the hypotheticals with those in *Bourhill* and *P2*. There is no way to represent either of the alternative theories under which *Bourhill* has greater relevant similarity to one hypothetical than to the other.

Figure 5: Featural and dimensional representations of precedents and hypotheticals.

A featural representation of the cases might be that each case consists of two features (foreseeability and directness) each with three possible values (the same as under the dimensional approach). The featural approach would differ from the dimensional approach in two respects. First, under the featural approach there would be no notion that changing a value feature in a particular direction would tend to establish or negate a given recovery by the claimant. Second, the featural approach would have explicit feature weighting.[7]

At first glance it appears that if such a weighting scheme could be devised, it would provide a mechanism for expressing the justification of individual precedents. For example, suppose that *Bourhill* and *P2* were decided under the first theory, under which *H1* should be controlled by *Bourhill* and *H2* should be controlled by *P2*. This could be modeled by placing a higher weight on the foreseeability feature than on the directness feature. If the distance function were $\sqrt{a|F_1 - F_2| + b|D_1 - D_2|}$, where F_n and D_n are the values (1–3) of foreseeablility and directness of consequences respectively for case n, a and b are the weights of wrongdoing and causation respectively, and adjacent feature values are separated by a unit distance, then setting $a > b$ would cause *H2*'s strongest match to be to *P2* and *H1*'s strongest match to be to *Bourhill*.

However, suppose that there is a third hypothetical, *H3*, in which the motorcylist passes on the side of the tram from which the Plaintiff is alighting, missing her by inches. At that moment, an IRA bomb goes off, injuring Plaintiff. *H3*, shown in Figure 5, differs from *P2* by a distance of 2 in its directness feature but not at all in its foreseeability feature. It differs from *Bourhill* by a distance of 1 in each feature. Thus, *H3*'s difference from *P2* is $\sqrt{a|2-2| + b|0-2|} = \sqrt{2b}$, and *H3*'s difference from *Bourhill* is $\sqrt{a|2-1| + b|0-1|} = \sqrt{a+b}$. *H3* clearly should have the same result as *Bourhill*, since there is no direct connection between the motorcyclist's wrongdoing and the Plaintiff's injury. Thus, it must be the case that the distance from *P2*, $\sqrt{2b}$, is greater than the distance from *Bourhill*, $\sqrt{a+b}$. However, $\sqrt{2b} > \sqrt{a+b}$ implies that $b > a$, which contradicts the assumption that $a > b$. Thus, no plausible global feature weighting scheme can adequately model the effect of the justifications of *Bourhill* and *P2* on these hypotheticals.

Finally, the transformation coherence approach suffers from the same inadequacy as the other approaches. Whether the transformations from *Bourhill* to *H1* and *P2* to *H2* are more or less coherent than the transformations from *Bourhill* to *H2* and *P2* to *H1* depends only on the facts being transformed and not at all on the theory under which *Bourhill* and *P2* were decided.[8] Thus, this approach is also inadequate, standing alone, to model the the effects of different possible *ratios* of the precedents.

In summary, exemplar-based reasoning is essential for any adequate model of *ratio decidendi*, but a model of *ratio* that consists only of exemplar-based reasoning implicitly subscribes to Goodhart's view and is therefore subject to Cross's critique.

5 Conclusion

This paper has proposed a model of *ratio decidendi* under which the authoritative elements of a precedent include (1) each warrant for the ultimate result appearing in the reduction graph of the precedent's justification and (2) each reduction operator connecting successive warrants in

[7] The infeasibility of devising such a weighting scheme is discussed in [AR88].

[8] A recent description of the prototypes-and-deformations model suggests that it may now be intended to apply to the explanations, as well as the facts, of precedents [McC91]. This refinement would represent a departure from Goodhart's view.

this reduction graph. This collection comprises the warrants in the precedent's justification that satisfy Wambaugh's criterion: if any such warrant were false, then the decision would no longer follow from the facts of the case. This model satisfies four adequacy criteria for *ratio decidendi* identified from the jurisprudential literature. In particular, the model shows how the theory under which a case is decided controls its precedential effect. By contrast, a purely exemplar-based model of *ratio* implicitly adheres to Goodhart's view of *ratio* and therefore suffer from its critical weakness: failure to represent the reasoning under which a decision follows from the material facts of the case.

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